1. For each of the functions below, describe the domain of definition, and write each function in the form $f(z) = u(x, y) + iv(x, y)$.

   1) $f(z) = \dfrac{z^2}{z + \overline{z}}$

   2) $f(z) = z^3$

   3) $f(z) = |z| + \overline{z}$

2. Suppose that $f(z) = x^2 - y^2 - 2y + i(2x - 2xy)$, where $z = x + iy$. Use the expressions

$$x = \frac{z + \overline{z}}{2} \quad \text{and} \quad y = \frac{z - \overline{z}}{2i}$$

   to write $f(z)$ in terms of z, and simplify the result.

3. Sketch the following sets and determine which are domains. If the set is NOT a domain, briefly explain why not.

   1) $1 < |z - i| < 4$

   2) $|z| < 1$ or $|z| > 2$

   3) $\operatorname{Re} z \geq 2$

   4) $-\pi < \arg z < \pi \ (z \neq 0)$

4. Evaluate the following limit. Show steps. If the limit does not exist, explain why not.

   1) $\displaystyle\lim_{z \to i} \frac{z^3 + 3iz^2 + z + 3i}{z - i}$

   2) $\displaystyle\lim_{z \to 0} \frac{z}{|z|}$

   3) $\displaystyle\lim_{z \to \infty} \frac{4z^2}{(z - 1)^2}$

   4) $\displaystyle\lim_{z \to \infty} \frac{z^2 + 1}{z - 1}$

5. Let

$$f(z) = \begin{cases} \dfrac{\operatorname{Im}(z^2)}{2i|z|} & \text{if } z \in \mathbb{C}\backslash\{0\}; \\ w & \text{if } z = 0. \end{cases}$$

   Determine the value w so that the function f is continuous at $z = 0$.

6. Let $f(x + iy) = e^x(\cos y + i \sin y)$ for all $x, y \in \mathbb{R}$. Prove that f is continuous on the complex plane.