2-2 *Group velocity of localized waves*. Section 2-2 discusses the superposition of *two* classical sinusoidal waves of slightly different wavenumbers k and $k + \Delta k$ to yield a modulated result whose modulations move with a group velocity. A composition almost as simple as the two-component case is the superposition of *three* waves y_0, y_1, and y_2 with the same total spread Δk in wavenumber ($k = 2\pi/\lambda$):

$$y_0 = A \sin (kx - \omega t)$$
$$y_1 = \frac{A}{2} \sin \left[\left(k - \frac{\Delta k}{2}\right) x - \left(\omega - \frac{\Delta \omega}{2}\right) t\right]$$
$$y_2 = \frac{A}{2} \sin \left[\left(k + \frac{\Delta k}{2}\right) x - \left(\omega + \frac{\Delta \omega}{2}\right) t\right]$$

(The amplitude $A/2$ of y_1 and y_2 has been chosen to give the superposition the simplest possible form.)

(a) Express the superposition $y_o + y_1 + y_2$ as a single product of trigonometric functions. [*Hint*: Add $y_1 + y_2$ first and use trigonometric identities.] Sketch the resultant wave for $t = 0$.

(b) If $\Delta k/k = 10^{-2}$, how many zeros does the waveform have within each region of reinforcement (between adjacent zeros of the envelope)?

(c) If the phase velocity $\omega/k = 10$ cm/sec, $\Delta k/k = 10^{-2}$, and $\Delta\omega/\omega = 10^{-3}$, then what is the group velocity of the waveform?