20. Verify both algebraically and geometrically that the following properties of vector arithmetic hold. (Do so for $n = 2$ if the general case is too intimidating.)

a. For all $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, $\mathbf{x} + \mathbf{y} = \mathbf{y} + \mathbf{x}$.

b. For all $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{R}^n$, $(\mathbf{x} + \mathbf{y}) + \mathbf{z} = \mathbf{x} + (\mathbf{y} + \mathbf{z})$.

c. $\mathbf{0} + \mathbf{x} = \mathbf{x}$ for all $\mathbf{x} \in \mathbb{R}^n$.

d. For each $\mathbf{x} \in \mathbb{R}^n$, there is a vector $-\mathbf{x}$ so that $\mathbf{x} + (-\mathbf{x}) = \mathbf{0}$.

e. For all $c, d \in \mathbb{R}$ and $\mathbf{x} \in \mathbb{R}^n$, $c(d\mathbf{x}) = (cd)\mathbf{x}$.

f. For all $c \in \mathbb{R}$ and $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, $c(\mathbf{x} + \mathbf{y}) = c\mathbf{x} + c\mathbf{y}$.

g. For all $c, d \in \mathbb{R}$ and $\mathbf{x} \in \mathbb{R}^n$, $(c + d)\mathbf{x} = c\mathbf{x} + d\mathbf{x}$.

h. For all $\mathbf{x} \in \mathbb{R}^n$, $1\mathbf{x} = \mathbf{x}$.

21. a. Using only the properties listed in Exercise 20, prove that for any $\mathbf{x} \in \mathbb{R}^n$, we have $0\mathbf{x} = \mathbf{0}$. (It often surprises students that this is a consequence of the properties in Exercise 20.)

b. Using the result of *a*, prove that $(-1)\mathbf{x} = -\mathbf{x}$. (Be sure you didn't use this fact in your proof of *a*!)