1. Suppose f and g are in $L^2((0,2\pi))$ and are periodic of period 2π and let $S[f] = \sum_{k=-\infty}^{\infty} c_k e^{ikx}$ and let $S[g] = \sum_{k=-\infty}^{\infty} d_k e^{ikx}$. We define the convolution of f and g by

$$f * g(x) = \frac{1}{2\pi} \int_{(0,2\pi)} f(x-t)g(t)\ dt.$$

Show that $S[f*g] = \sum_{k=-\infty}^{\infty} c_k d_k e^{ikx}$ and show that $f*g(x) = \sum_{k=-\infty}^{\infty} c_k d_k e^{ikx}$ where the series converges in $L^2((0,2\pi))$.

2. Let $f(x) = \frac{1}{2}(\pi - x)$ for $0 < x < 2\pi$ and assume that f is periodic of period 2π. Find the Fourier series for f and show that

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}\ .$$

Use the previous problem to find the value of the series

$$\sum_{k=1}^{\infty} \frac{1}{k^4} = \frac{\pi^4}{90}\ .$$