1. Let ϕ be a continuous function on $(0, \infty)$. Define a function on $\mathbb{R}^n$ by $\Phi(x) = \phi(|x|)$.
Let $0 < a < b < \infty$ and show that

$$\int_{a<|x|<b} \Phi(x) = n\Omega_n \int_a^b \phi(t) t^{n-1}\ dt,$$

where Ω_n is the volume of the unit ball in $\mathbb{R}^n$.
Hint: Use the following lemma and\or its proof:

> Suppose that $f(x)$ is integrable over a spherical shell $a \leq |x| \leq b$ and that $\phi(\rho)$ is continuous for $a \leq \rho \leq b, 0 \leq a < b < \infty$. Let $F(\rho) = \int_{a \leq |x| \leq \rho} f(x)\ dx$ for $a \leq \rho \leq b$. Then
>
> $$\int_{a \leq |x| \leq b} f(x)\phi(|x|)\ dx = \int_a^b \phi(\rho) dF(\rho)$$
>
> the integral on the right being a Riemann stieltjes integral.

2. For α in $\mathbb{R}$ and $R > 0$ find the value of the integral

$$\int_{|x|<R} |x|^\alpha\ dx.$$