Let *n* be a positive integer. Let A be an element of the vector space *Mat(n,n,****F****)*, which has dimension *n2* over **F**. Show that the span of the infinite set of matrices *span*(In, A, A2, A3, …) has dimension not exceeding *n* over **F**.

Defn of the linear space *Mat(n,n,****F****)*:

The set of all *n*-by-*n* matrices with entries in **F.** *Mat(n,n,****F*** *)* is a vector space. The additive identity in *Mat(n,n,****F*** *)* is the *n*-by-*n* matrix all of whose entries equal zero.

Clues to solving the problem:

A is an element of *Mat(n,n,****F*** *)*

dim *Mat(n,n,****F*** *)* = n2

In = Identity matrix. 1’s on the diagonal and 0’s else.

In, A, A2, … , An^2 -- n+1 matrices are linearly dependent.

Want to show that *dim span*(In, A, A2, A3, …), all infinitely many of them, are less than or equal to *n*.

**Similar problem:**

Assume that A is invertable (that is that A-1 exists) then A-1 is an element of *span*(In, A, A2, A3, …,An-1).

Reformulation: A-1 = p(A) for a polynomial P

Proof: Interpret A as M(T) for some *T* in L(**F**n). let c be the minimum polynomial for T.

c = a0 +a1z+ … + amzm, m ≤ n

means that c(T) = 0, assume am ≠ 0

a0 +a1T + … + amTm = 0 If a0 = 0, then c(0)=0, and 0 is an eigenvalue for T. T is not injective and A-1 does not exist. Since it DOES exist(due to the hypothesis), c(0)≠0, therefore a0 ≠ 0

So, 1 = -(a1/a0)T - … - (am/a0)Tm

So, In = -(a1/a0)A - … - (am/a0)Am

So, A-1 = -(a1/a0) In - … - (am/a0)Am-1