1. Imagine that you are hired as a supply chain expert to develop a linear programming formulation that is a mathematical representation (or estimate) of a classic business challenge for a large company that manufactures products that are sold directly to its customers. Through this exam exercise, you will start with a very simple problem and build complexity as you go through each of the below. Assume that there is only one stage of production to make each product and that you start with the following basic data sets:

   PROD = list of different products that the company sells
   FAC = list of different factories that the company owns and runs to manufacture products
   LIN = list of different types of manufacturing lines in the different factories i.e., L1 could be a newer, more efficient line type while L2 could be a line type that can only make certain types of products. The company could have 5 L1s in one factory and only 2 in another factory
   TIM = list of planning time periods
   CUST = list of customers who buy products from the company
   CUSTLOC = list of customer locations where products are to be shipped

   i. If the company does not hold inventory or allow backlogging of demand (late) and didn't have any manufacturing capacity restrictions, what decision variables and constraints would be needed to help the company decide what products to make in which factories to satisfy customer demands for products? Assume that you have the following demand data and that all products can be made at all factories.
      DEMAND(TIM, PROD, CUST) = units of product demanded by customer in each period

   ii. Suppose that some lines yield a higher percentage of good products than others and that this information is in a table
      YLD(FAC,LIN, PROD) = Yield of good product off a given line type in a given factory
      Since the factories may maintain and configure the lines differently, the yields from one factory to another with the same kind of line may differ. How do the decision variables and constraints in i) change if the company wants to incorporate these yields into the model?

   iii. Suppose now that the company has capacity restrictions to consider and allows that some demand is met late. That is, if not all demand for a period could be met, they could 'roll' that (and any other previously built backlog) unmet demand into the next period. Associated data tables now provided (along with the above) include:
      RATE(LIN, PROD) = hours of time required to make one unit of a product on a line type
      CAP(TIM,FAC,LIN) = hours of time available at a factory for a line type in each period
      Would the decision variables and constraints in i) and ii) above need to be defined differently? What new decision variables and constraints would need to be added to the formulation?

iv. Thankfully, the company's factories are co-located with buildings where they can store and pre-build product inventory if needed in an attempt to avoid backlogging. Assume that you are now additionally given the following inventory snapshot to start with INITINV(FAC,PROD) = units of a product currently in inventory at a factory. What decision variables and constraint modifications are needed to reflect this element of their business (that they can have/hold inventory)?

v. Based on historical sales and shortages, the company would additionally like hold a minimum safety stock of inventory but prefers to meet demands on-time and let inventory levels drop below this threshold if needed. They provide the following inventory target data: MININV(TIM,FAC, PROD) = units of a product to target having in inventory at factory at the end of each period . What additional decision variables and/or constraint modifications or new constraints are needed to have the model incorporate this business practice?

vi. The company would now like to have some planning insight into their future logistics need to move product from factories to customers. Factories only ship products directly to the customer locations. To avoid having a model that allows for 'silly' shipments (i.e., they would never ship from a California factory to a New York customer location), the company has provided the below information about shipping allowances:
SHIPMAP(FAC,CUSTLOC) = indication of which factories can ship to which customer locations (any product)
Assuming that shipments leave factories and arrive at the customer locations in the same period, what are the new decision variables and additional and/or modified constraints to include shipping concerns into the formulation? Assume that the DEMAND table is now dimensioned on TIM, PROD and CUSTLOC.

vii. Everything so far has focused on the decision variables and constraints. What would you guess the objective function would look like and what data would be needed to support this (define specific tables & definitions as I have done throughout this question)?

Extra:

Suppose that the company now indicates that it would like the model to be driven according to a customer priority practice. That is, it has customer tiers defined such that the highest tier customers should have their orders/demand met before other all other customers while the lowest tier customer demands should be met only after all other customer tiers are met. What data would you need from the company to incorporate this and how would you model it (modifications or new variables/constraints)? What would the objective function need to look like to drive the model to align with this business approach?

As an FYI.. some additional common extensions to the above include: incorporation of specific line information within each factory and an availability/maintenance calendar, labor concerns, product introduction/phase-out, time phased product/factory introduction/phase outs and yields, dynamic shipping allowances (if you're really in a crunch and you need shipment beyond normal shipping routes, let the model 'open' up additional ones at a higher yet reasonable cost, maximum inventory capacities by factories and allowance of shipments between factories, maximum number of periods that backlogged demand will stay 'alive' (customer bails/goes elsewhere), perishable product concerns so inventory can last just long, shipping takes longer so doesn't leave and arrive in same period, multi-period manufacturing time (i.e., it takes 3 periods to make one product and only 2 periods to make another) and so on and so on! ☺.