2. If a vector $\mathbf{Y}_3$ lies in the plane determined by the two vectors $\mathbf{Y}_1$ and $\mathbf{Y}_2$, then we can write $\mathbf{Y}_3$ as a linear combination of $\mathbf{Y}_1$ and $\mathbf{Y}_2$. That is,

$$\mathbf{Y}_3 = k_1\mathbf{Y}_1 + k_2\mathbf{Y}_2$$

for some constants k_1 and k_2. But then

$$k_1\mathbf{Y}_1 + k_2\mathbf{Y}_2 - \mathbf{Y}_3 = (0, 0, 0).$$

Show that if

$$k_1\mathbf{Y}_1 + k_2\mathbf{Y}_2 + k_3\mathbf{Y}_3 = (0, 0, 0),$$

with not all of k_1, k_2, and $k_3 = 0$, then the vectors are not linearly independent. [*Hint*: Start by assuming that $k_3 \neq 0$ and show that $\mathbf{Y}_3$ is in the plane determined by $\mathbf{Y}_1$ and $\mathbf{Y}_2$. Then treat the other cases.] Note that this computation leads to the theorem that three vectors $\mathbf{Y}_1$, $\mathbf{Y}_2$, and $\mathbf{Y}_3$ are linearly independent if and only if the only solution of

$$k_1\mathbf{Y}_1 + k_2\mathbf{Y}_2 + k_3\mathbf{Y}_3 = (0, 0, 0)$$

is $k_1 = k_2 = k_3 = 0$.

3. Using the technique of Exercise 2, determine whether or not the following sets of three vectors are linearly independent.

(a) $(1, 2, 1)$, $(1, 3, 1)$, $(1, 4, 1)$

(b) $(2, 0, -1)$, $(3, 2, 2)$, $(1, -2, -3)$

(c) $(1, 2, 0)$, $(0, 1, 2)$, $(2, 0, 1)$

(d) $(-3, \pi, 1)$, $(0, 1, 0)$, $(-2, -2, -2)$

In Exercises 4–7, consider the linear system $d\mathbf{Y}/dt = \mathbf{AY}$ with the coefficient matrix $\mathbf{A}$ specified. Each of these systems decouples into a two-dimensional system and a one-dimensional system. For each exercise,

(a) compute the eigenvalues,

(b) determine how the system decouples,

(c) sketch the two-dimensional phase plane and one-dimensional phase line for the decoupled systems, and

(d) give a rough sketch of the phase portrait of the system.

4. $\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

5. $\mathbf{A} = \begin{pmatrix} -2 & 3 & 0 \\ 3 & -2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

6. $\mathbf{A} = \begin{pmatrix} 1 & 0 & 3 \\ 0 & -1 & 0 \\ -3 & 0 & 1 \end{pmatrix}$

7. $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$