

1. Classify each of the following system of differential equations as either sink, source, saddle, center, spiral sink, or spiral source.

$$(a) \frac{dY}{dt} = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} Y$$

$$(b) \frac{dY}{dt} = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix} Y$$

$$(c) \frac{dY}{dt} = \begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix} Y$$

$$(d) \frac{dY}{dt} = \begin{pmatrix} 1 & 2 \\ -2 & -1 \end{pmatrix} Y$$

2. (a) Find the general solution of $\frac{d^2y}{dt^2} + 9y = 0$

(b) Find the general solution of $\frac{d^2y}{dt^2} + 9y = 27\cos 2t$

(c) For the equation in part (b), find the particular solution if $y(0) = y'(0) = 0$

(d) For the solution in part (c), find the frequency of the beats and frequency of fast oscillations.

(e) Describe the long term behavior of

$$\frac{d^2y}{dt^2} + 9y = \cos 3t$$