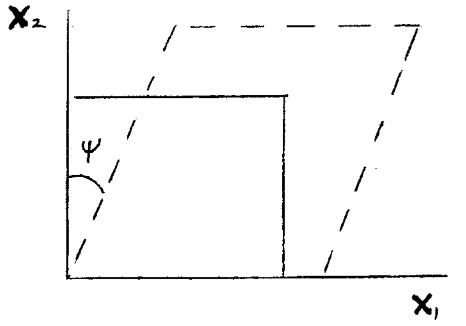
Problem C

Consider the following spatially homogeneous deformation of a material.

Notice: Capital "X"
lowercase "x"



$$x_1 = \delta X_1 + \lambda X_2 \sin\psi$$

$$x_2 = \beta X_2 \cos\psi$$

$$x_3 = \beta X_3$$

$\underset{\sim}{E}$ = tensor
$\underline{N}' = \underline{e}_2$: (example of vectors)

a) Find the Lagrangian strain tensor, $\underset{\sim}{E}$, and the small strain tensor, $\underset{\sim}{\epsilon}$, in terms of δ, β, ψ.

b) For $\delta = 1.66$, beta $\beta = 1.55$, $\psi = 30°$

i) Determine ϵ_E for a differential vector

$\underline{N} = \frac{1}{\sqrt{2}}(\underline{e}_1 + \underline{e}_2)$ using finite strain and small strain tensor calculations.

ii) Determine the angle change for two differential vectors oriented as

$$\underline{N}^1 = \underline{e}_2 \quad \& \quad \underline{N}^2 = \frac{\sqrt{3}}{2}\underline{e}_1 - \frac{1}{2}\underline{e}_2$$

using finite strain only.

iii) Determine the angle change, $\Delta\theta$, for two differential vectors oriented as

$$\underline{N}' = \underline{e}_1 \quad , \quad \underline{N}_2 = \underline{e}_2$$

using finite strain as well as the small strain approximation given by

$$\Delta\theta = \underline{N}_1 \cdot \underset{\sim}{\epsilon}\,\underline{N}_2 .$$

iv) Determine the volumetric strain for finite strain and small strain theory.

c) Repeat (b) for $\delta = 1.04$, $\beta = 1.03$ $\theta = 2°$.

Caution: Watch upper and lowercase notation