LAB 2.1 Two Magnets and a Spring

In this lab we consider the motion of a mass that can slide freely along the x-axis. The mass is attached to a spring that has its other end attached to the point $(0, 2)$ on the y-axis. In addition, the mass is made of iron and is attracted to two magnets of equal strength—one located at the point $(-1, -a)$ and the other at $(1, -a)$ (see Figure 2.64).

We assume that the spring obeys Hooke's Law, and the magnets attract the mass with a force proportional to the inverse of the square of the distance of the mass to the magnet (the inverse square law). If we choose the spring constant, mass, strength of the magnets, and units of distance and time appropriately, then we can model the motion of the mass along the x-axis with the equation

$$\frac{d^2x}{dt^2} = -0.3x - \frac{x-1}{((x-1)^2 + a^2)^{3/2}} - \frac{x+1}{((x+1)^2 + a^2)^{3/2}}.$$

(A good exercise for engineering and physics students: Derive this equation and determine the units and choices of spring constant, rest length of the spring, mass, and strength of the magnets involved.)

The goal of this lab is to study this system numerically. Use technology to find equilibria and study the behavior of solutions. Be careful to consider the correct regions of the phase plane at the correct scale so that you can find the important aspects of the system.

In your report, you should address the following items:

1. Consider the system with the parameter value $a = 2.0$. Discuss the behavior of solutions in the phase plane. Relate the phase portrait to the possible motions of the mass along the x-axis.

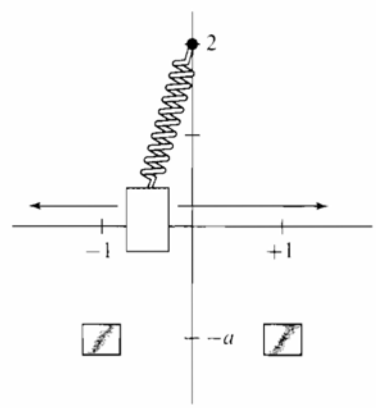

Figure 2.64
Schematic of a mass sliding on the x-axis attached to a spring and attracted by two magnets.

2. Consider the system with the parameter value $a = 0.5$. Discuss the behavior of solutions in the phase plane. Relate the phase portrait to the possible motions of the mass along the x-axis. Be particularly careful to describe the solutions that separate different types of qualitative behavior.

3. Describe how the system changes as a varies from $a = 0.5$ to $a = 2.0$. That is, describe the bifurcation that occurs.

4. Finally, repeat the analysis in Parts 1–3 with the magnets located at $(\pm 2, -a)$. In other words, use the equation

$$\frac{d^2x}{dt^2} = -0.3x - \frac{x-2}{((x-2)^2 + a^2)^{3/2}} - \frac{x+2}{((x+2)^2 + a^2)^{3/2}}.$$

Note the differences between this system and the previous one and interpret these differences in terms of the possible motions of the mass as it slides along the x-axis.

Your report: Address each of the items above. Pay particular attention to the physical interpretation of the solutions in terms of the possible motions of the mass as it slides along the x-axis. You may include graphs and phase portraits to illustrate your discussion, but pictures alone are not sufficient.

LAB 4.1 Two Magnets and a Spring Revisited

In this lab we again study the motion of a mass that can slide on the x-axis (see Lab 2.1). The mass is attached to a spring that has its other end attached to the point $(0, 2)$ on the y-axis. In addition, the mass is made of iron and is attracted to two magnets of equal strength—one located at the point $(-1, -a)$ and the other at $(1, -a)$ (see Figure 4.40).

We assume that the spring obeys Hooke's Law, and the magnets attract the mass with a force proportional to the inverse of the square of the distance of the mass to the magnet (the inverse square law). In Lab 2.1, we observed a subtle dependence of the solutions on the position of the magnets.

In this lab, we consider the effect of an external forcing term on this system. We can think of the external force as a wind that gusts, alternately blowing the mass to the left and the right. More precisely, we study the solutions of the nonautonomous second-order equation

$$\frac{d^2x}{dt^2} = -0.3x - \frac{x-1}{((x-1)^2 + a^2)^{3/2}} - \frac{x+1}{((x+1)^2 + a^2)^{3/2}} + b\cos t,$$

where b is the amplitude of the forcing. This equation is very complicated, so you are expected to carry out your analysis numerically. In order to observe the effects of the forcing, you will have to follow the solutions over intervals of time that are at least as long as several periods of the forcing function.

Address the following items in your report:

1. Recall and summarize the behavior of the unforced system ($b = 0$), the one you studied in Lab 2.1. Briefly describe both the phase portraits and the motion of the mass along the x-axis for the two positions of the magnets $a = 0.5$ and $a = 2$.

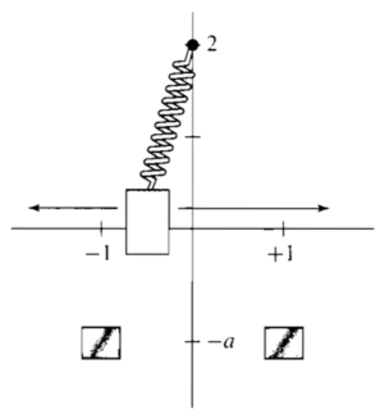

Figure 4.40
Schematic of a mass sliding on the x-axis attached to a spring and attracted by two magnets.

2. Study solutions to the forced system for magnets that are far from the x-axis assuming the amplitude of the forcing is small, for example, $a = 2$ and $b = 0.1$. How do solutions differ from those in Part 1? Express your conclusions in terms of the solution curves in the phase plane and in terms of the motion of the mass along the x-axis.

3. Study solutions of the forced system for magnets that are close to the x-axis assuming the amplitude of the forcing is small, for example, $a = 0.5$ and $b = 0.1$. How do solutions differ from the unforced case? Express your conclusions in terms of the solution curves in the phase plane and in terms of the motion of the mass along the x-axis. Pay particular attention to solutions whose initial conditions are near the origin in the phase plane.

In your report, pay particular attention to the physical interpretation of the solutions in terms of the possible motions of the mass as it slides along the x-axis. Include graphs and phase portraits to illustrate your discussion, but pictures alone are not sufficient.