# Bifurcations in Linear Systems

In Chapter 3, we have studied techniques for solving linear systems. Given the coefficient matrix for the system, we can use these techniques to classify the system, describe the qualitative behavior of solutions, and give a formula for the general solution. In this lab we consider a two-parameter family of linear systems. The goal is to better understand how different linear systems are related to each other, or in other words, what bifurcations occur in parameterized families of linear systems.

Consider the linear system

$$\frac{dx}{dt} = ax + by$$
$$\frac{dy}{dt} = -x - y,$$

where $a$ and $b$ are parameters that can take on any real value. In your report, address the following items:

1. For each value of $a$ and $b$, classify the linear system as source, sink, center, spiral sink, and so forth. Draw a picture of the $ab$-plane and indicate the values of $a$ and $b$ for which the system is of each type (that is, shade the values of $a$ and $b$ for which the system is a sink red, for which it is a source blue, and so forth). Be sure to describe all of the computations involved in creating this picture.

2. As the values of $a$ and $b$ are changed so that the point $(a, b)$ moves from one region to another, the type of the linear system changes, that is, a bifurcation occurs. Which of these bifurcations is important for the long-term behavior of solutions? Which of these bifurcations corresponds to a dramatic change in the phase plane or the $x(t)$- and $y(t)$-graphs?

**Your report:** Address the items above in the form of a short essay. Include any computations necessary to produce the picture in Part 1. You may include phase planes and/or graphs of solutions to illustrate your essay, but your answer should be complete and understandable without the pictures.

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