Prove: That for resistors in parallel configuration, the equivalent resistance is always smaller than the smallest resistor.

Proof:

Let's start with two resistors: R1 and R2.

We know that:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

This becomes:

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Now, lets assume that R_2 is smaller of the two resistors. We can rewrite the equation as:

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{R_1 R_2}{R_1\left(1 + \frac{R_2}{R_1}\right)} = \frac{R_2}{\left(1 + \frac{R_2}{R_1}\right)}$$

But since resistance is a positive number we can say that $\left(1 + \frac{R_2}{R_1}\right) > 1$ thus:

$$R_{eq} = \frac{R_2}{\left(1 + \frac{R_2}{R_1}\right)} < R_2$$

We can generalize this to n resistors, by building our network piece-by-piece, adding one resistor at a time.
Of-course we can also use brute force:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots \frac{1}{R_n}$$

$$R_{eq} = \frac{R_1 R_2 \cdots R_n}{\left(R_2 R_3 \ldots R_n\right) + \left(R_1 R_3 \ldots R_n\right) + \ldots + \left(R_1 R_2 \ldots R_{n-1}\right)}$$

As before, let's designate the smallest resistor as R_1. We can rewrite the above equation as:

$$R_{eq} = \frac{R_1(R_2R_3\dots R_n)}{(R_2R_3\dots R_n)+(R_1R_3\dots R_n)+\dots+(R_1R_2\dots R_{n-1})} =$$

$$= \frac{R_1(R_2R_3\dots R_n)}{(R_2R_3\dots R_n)\left[1+\frac{R_1}{R_2}+\frac{R_1}{R_3}+\cdots+\frac{R_1}{R_n}\right]} = \frac{R_1}{\left[1+\frac{R_1}{R_2}+\frac{R_1}{R_3}+\cdots+\frac{R_1}{R_n}\right]} < R_1$$

QED