A series $\sum_{k=0}^{\infty} a_k$ is said to be *Cesaro summable* to L if

$$\sigma_n := \sum_{k=0}^{n-1} \left(1 + \frac{k}{n}\right) a_k$$

converges to L as $n \to \infty$.

a) Let $s_n = \sum_{k=0}^{n-1} a_k$. Prove that

$$\sigma_n := \frac{s_1 + s_2 + \cdots + s_n}{n}$$

for each $n \in \mathbb{N}$.

b) Prove that if $a_k \in \mathbb{R}$ and $\sum_{k=0}^{\infty} a_k = L$ converges, then $\sum_{k=0}^{\infty} a_k$ is Cesaro summable to L.

c) Prove that $\sum_{k=0}^{\infty} (-1)^k$ is Cesaro summable to 1/2; hence the converse of b) is false.

d) Prove that if $a_k \geq 0$ for $k \in \mathbb{N}$ and $\sum_{k=0}^{\infty} a_k$ is Cesaro summable to L, then $\sum_{k=0}^{\infty} a_k = L$.

Since this problem is an analysis problem, please be sure to be rigorous.