Question 2 – Parabolic PDE

Find in 0 < x' <a, t'>0, the Green's function G(x, t, x', t') satisfying equations 6.18 and 6.20 and the additional conditions $G_{x'}(x, t, a, t') = G(x, t, 0, t') = 0$.

Equation 6.18 is $P'_3 G = \delta(x' - x)\delta(y' - y)$, where $P'_3 = u_{xx} + u_t$ is the **adjoint** of the heat equation.

Equation 6.20 is $G \equiv 0$, t'>t

The answer is given as:

$$-\frac{2}{a}\sum_{n=1}^{\infty} exp\left\{\frac{(2n-1)^2}{4a^2}\pi^2(t'-t)\right\}\sin(2n-1)\frac{\pi x'}{2a}\sin(2n-1)\frac{\pi x}{2a},\ t' < t$$