Let K be an nxn matrix and λ a small number. Imitating

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$$

valid for small x, it is natural to define

$$\log(1+\lambda K) = \lambda K - \frac{\lambda^2}{2}K^2 + \frac{\lambda^3}{3}K^3 - \cdots$$

Explain why this makes sense. Prove

$$\operatorname{trace}\log(1+\lambda K) = \log\det(I+\lambda K)$$

Still with λ small so that everything makes sense. Hint: What is $I - \lambda K + \lambda^2 K^2 - \lambda^3 K^3 + \cdots$?

Since this is an analysis problem, please be sure to be rigorous, and include as much detail as possible so that I can understand. Please also state if you are making use of some fact or theorem. Thanks!