

Consider the sphere $x_0^2 + x_1^2 + \cdots + x_n^2 = n$ of radius \sqrt{n} . Show that the normalized volume of the spherical band where $a \leq x_0 \leq b$ is

$$v_n = \frac{\int_a^b (n - x_0^2)^{n/2} dx_0}{\int_{-\sqrt{n}}^{\sqrt{n}} (n - x_0^2)^{n/2} dx_0}$$

and prove that

$$\lim_{n \uparrow \infty} v_n = \int_a^b \frac{e^{x_0^2/2}}{\sqrt{2\pi}}$$

Hint: $1 - x \leq e^{-x}$ will be helpful at one point.

Since this is an analysis problem, please be sure to be rigorous, and include as much detail as possible so that I can understand. Please also state if you are making use of some fact or theorem. Thanks!