

Let K be an $n \times n$ matrix and λ a small number. Imitating

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$$

valid for small x , it is natural to define

$$\log(1 + \lambda K) = \lambda K - \frac{\lambda^2}{2} K^2 + \frac{\lambda^3}{3} K^3 - \dots$$

Explain why this makes sense. Prove

$$\text{trace } \log(1 + \lambda K) = \log \det(I + \lambda K)$$

Still with λ small so that everything makes sense. Hint: What is $I - \lambda K + \lambda^2 K^2 - \lambda^3 K^3 + \dots$?

Since this is an analysis problem, please be sure to be rigorous, and include as much detail as possible so that I can understand. Please also state if you are making use of some fact or theorem. Thanks!