

## Generalized functions problems.

① Show that in any strip  $-n-1 < \operatorname{Re} \lambda < -n$

$$\text{that } \langle x_+^\lambda, \phi \rangle = \int_0^\infty x^\lambda [\phi(x) - \phi(0) - \dots - \frac{x^{n-1}}{(n-1)!} \phi^{(n-1)}(0)] dx$$

② Show that  $|x|^\lambda = x_+^\lambda + x_-^\lambda$ . Construct the regularization of  $|x|^\lambda$  following from this relation. Using ① and its analogue for the generalized function  $x_-^\lambda$  show that for  $-2m-1 < \operatorname{Re} \lambda < -2m+1$  that

$$\begin{aligned} \langle |x|^\lambda, \phi \rangle &= \int_0^\infty x^\lambda [\phi(x) - \phi(-x)] \\ &- 2 \left\{ \phi(0) + \frac{x^2}{2!} \phi^{(2)}(0) + \dots + \frac{x^{2m-2}}{(2m-2)!} \phi^{(2m-2)}(0) \right\} dx \end{aligned}$$

where are the poles of the Generalised function  $|x|^\lambda$  in the complex plane and what are their residues. Show that  $\frac{|x|^\lambda}{\Gamma(\frac{\lambda+1}{2})}$  is an entire function of  $\lambda$ .

$$\frac{4z}{2z+1}$$