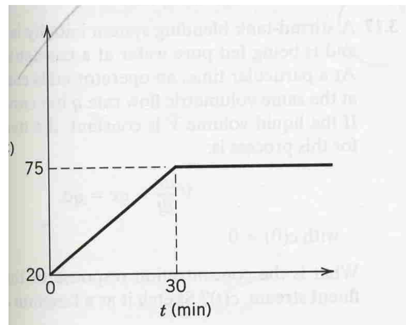
Problem 1

Consider the following transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{10}{5s + 1}$$

(a) What is the steady state gain and time constant?
(b) If U(s) =2/s, what is the value of the output when t→∞
(c) For the same U(s) what is the value of the output when t = 10?
(d) If U(s) is a unit rectangular pulse what is the output when t→∞?
(e) If u(t) is unit impulse at t=0 what is the output when t→∞?
(f) If u(t) = 2 sin(3t) what is the output when t→∞?

Problem 2



A stirred batch chemical reactor has the jacket temperature raised with the profile shown (final temperature is held indefinitely).
The change in the exit concentration of a product P, c_p' is related to the change in the jacket temperature (T_j') through the transfer function,

$$G(s) = \frac{0.04}{20s+1} = \frac{c_p'(s)}{T_J'(s)}$$

time constant in minutes,
gain is mol/L/°C
c_p', T_j' are deviation variables.

(a) The initial operating condition of the reactor is 20°C for both the jacket and the reactor temperatures, and an exit concentration c_P of 0.20 mol/l. Calculate the final concentration of P at the new steady state for the temperature change shown in the Figure.
(b) Would the steady state concentration be different if an input step change from 20 to 75°C occurred (instead of the initial ramp)? Why or why not?
(c) Suppose after *t* = 400 minutes, the input temperature returns to 20°C in a stepwise fashion. Sketch the behavior of $c_P(t)$.
(d) How long will it take for $c_P(t)$ to reach $c_P = 0.4$ after the input temperature returns to 20°C? Assume an initial condition of $c_P = 2.4$.

Problem 3

A heated process is used to heat a semiconductor wader operates with first-order dynamics, that is the transfer function relating changes in temperature T to changes in the heater input power level P is

$$\frac{T'(s)}{P'(s)} = \frac{K}{\tau s+1}$$ where K has units [°C/kw] and τ has units [minutes].

The process is at steady state when an engineer changes the power input stepwise from 1 to 1.5 kw. She notes the following:

(i) The process temperature initially is 80 °C.
(ii) Four minutes after changing the power input, the temperature is 230 °C.
(iii) Thirty minutes later the temperature is 280 °C.

(a) What are K and T in the process transfer function?
(b) If at another time the engineer changes the power input linearly at a rate of 0.5 kW/min, what can you say about the maximum rate of change of process temperature: When will it occur? How large will it be?

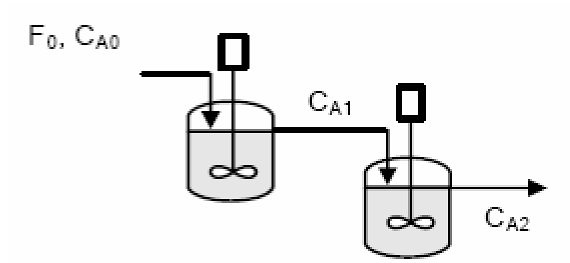
Problem 4

For $Y(s) = \dfrac{2}{s^3 + 6s^2 + 10s}$

(a) Sketch the function y(t) and identify y(0) and y(∞) on the graph of y vs. t.
(b) Is y (t) converging (stable) or diverging (unstable)? Is y (t) smooth or oscillatory? Explain your answers.
(c) Can you interpret Y(s) as the product of a process transfer function G(s) and a standard input change U(s)? What is G(s)?

Problem 5

Two stirred tanks are connected in series.



The dynamic process model for the tracer concentration as a function of time as given by the following equations (in deviation variables)

$$V_1 \frac{dC'_{A1}}{dt} = F_0 \left(C'_{A0} - C'_{A1} \right)$$
$$V_2 \frac{dC'_{A2}}{dt} = F_0 \left(C'_{A1} - C'_{A2} \right)$$

The operator instantaneously injects a large amount of tracer into the inlet of the first tank.
When does the peak tracer concentration occur (i.e., the maximum value of C_{A2})? Assume V_1=V_2.

Problem 6

The dynamic behavior of the liquid level in each leg of a manometer tube, responding to a change in pressure, is given by

$$\frac{d^2h'}{dt^2} + \frac{6\mu}{R^2\rho}\frac{dh'}{dt} + \frac{3}{2}\frac{g}{L}h' = \frac{3}{4\rho L}p'(t)$$

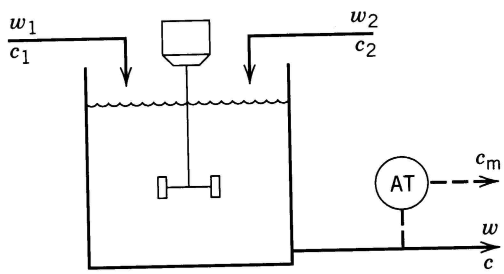
where h'{t) is the level of fluid measured with respect to the initial steady-state value, p'(t) is the pressure change, and R, L, q, p, and |JL are constants.

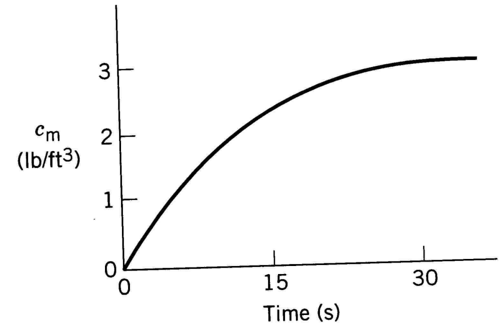
(a) Rearrange this equation into standard gain-time constant form and find expressions for K, T, I, in terms of the physical constants.
(b) For what values of the physical constants does the manometer response oscillate?
(c) Would changing the manometer fluid so that p (density) is larger make its response more or less oscillatory? Repeat the analysis for an increase in |JL (viscosity).

Problem 7

The caustic concentration of the mixing tank shown in the figure is measured using a conductivity cell. The total volume of solution in the tank is constant at 7 ft^3 and the density (p = 70 Ib/ft^3) can be considered to be independent of concentration. Let c_m denote the caustic concentration measured by the conductivity cell. The dynamic response of the conductivity cell to a step change (at t = 0) of 3 lb/ft^3 in the actual concentration (passing through the cell) is also shown in the figure.

(a) Determine the transfer function C'_m(s)/C'_1(s) assuming the flow rates are equal and constant: (w_1 = w_2 = 5 lb/min):
(b) Find the response for a step change in c_1 from 14 to 17 Ib/ft^3.
(c) If the transfer function C'_m{s)/C'(s) were approximated by 1 (unity), what would be the step response of the system for the same input change?
(d) By comparison of (b) and (c), what can you say about the dynamics of the conductivity cell? Plot both responses, if necessary.





Problem 8

A reactor process can be modeled as follows:

$$\left(16hr^2\right)\frac{d^2p(t)}{dt^2}+\left(3.2hr\right)\frac{dp(t)}{dt}+4p(t)+8psi=\left(0.008\frac{psi}{lbm/hr}\right)w(t)$$

Where: w(t) = reactor feed rate, lbm/hr

p(t) = reactor pressure, psia

The reactor is currently operating at steady state with w_s= 20 000 lbm/hr and p_s=38psia. The reactor is equipped with an automatic shutdown system that is activated anytime the reactor pressure reaches or exceeds 55 psia. The operator has been instructed to increase the federate to 25 000 lbm/hr. Assume the feed increase is performed in a single step (i.e. the feed rate is changed from 20 000 to 25 000 lbm/hr in a single, instantaneous step).

a) What is the reactor pressure after the system reaches a new steady state?

b) Will the reactor shut down? Justify your answer by predicting the maximum pressure that would be realized without a reactor shutdown system in place.