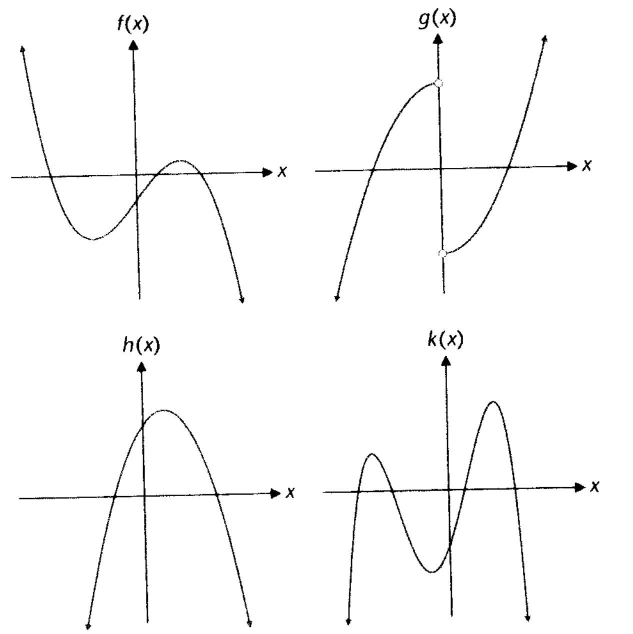
Problems 5–8 refer to the graphs of functions f, g, h, and k shown below.



5. Which of these functions could be a second-degree polynomial?

6. Which of these functions could be a third-degree polynomial?

7. Which of these functions could be a fourth-degree polynomial?

8. Which of these functions is not a polynomial?

In Problems 9–16, divide, using algebraic long division. Write the quotient, and indicate the remainder.

9. $(a^2 - 4) \div (a + 2)$ **10.** $(a^2 + 4) \div (a + 2)$

11. $(b^2 - 6b - 9) \div (b - 3)$

12. $(b^2 - 6b + 9) \div (b - 3)$

13. $(3x + 2 + x^3 - x^2) \div (x - 1)$

14. $(2x^2 + 4x - x^3 - 8) \div (x - 2)$

15. $(1 + 8y^3 - 4y^2 - 2y) \div (2y + 1)$

16. $(3 + 8y^3 - 6y^2 - 7y) \div (2y - 3)$

In Problems 17–22, use synthetic division to write the quotient $P(x) \div (x - r)$ in the form $P(x)/(x - r) = Q(x) + R/(x - r)$, where R is a constant.

17. $(x^2 + 4x - 15) \div (x - 3)$

18. $(x^2 - 2x - 1) \div (x - 4)$ **19.** $(3x^2 - x - 7) \div (x + 2)$

20. $(4x^2 + 18x + 4) \div (x + 5)$

21. $(2x^3 + 3x^2 - 8x + 1) \div (x + 3)$

22. $(3x^3 - 4x^2 - 7x + 9) \div (x - 2)$

B

Use synthetic division and the remainder theorem in Problems 23–28.

23. Find $P(-5)$, given $P(x) = 2x^2 + 8x - 6$.

24. Find $P(2)$, given $P(x) = 3x^2 - 4x + 2$.

25. Find $P(-4)$, given $P(x) = 4x^3 + 12x^2 - 8x + 25$.

26. Find $P(-3)$, given $P(x) = 5x^3 + 14x^2 + 3x + 10$.

27. Find $P(3)$, given $P(x) = 2x^4 - 5x^3 + 2x^2 - 11x - 14$.

28. Find $P(-6)$, given $P(x) = 3x^4 + 18x^3 + x^2 + 4x - 7$.

In Problems 29–44, divide, using synthetic division. Write the quotient, and indicate the remainder. As coefficients get more involved, a calculator should prove helpful. Do not round off—all quantities are exact.

29. $(2x^5 - 5x^2 + 3) \div (x - 1)$

30. $(3x^4 - 2x - 5) \div (x + 1)$

31. $(x^4 - 16) \div (x + 4)$ **32.** $(x^5 - 32) \div (x - 2)$

33. $(4x^4 - 9x^3 - 8x^2 - 2x - 7) \div (x - 3)$

34. $(2x^4 + 6x^3 - 4x^2 - 5x + 7) \div (x + 3)$

35. $(x^6 + 7x^5 + 10x^4 - x^2 - 5x) \div (x + 5)$

36. $(x^6 + 6x^5 + 2x^4 + 12x^3 - 3x - 18) \div (x + 6)$

37. $(2x^4 + 9x^3 + 5x^2 - 4x + 3) \div (x + \frac{3}{2})$

38. $(2x^4 + 5x^3 + 5x + 8) \div (x + \frac{1}{2})$

39. $(3x^4 + 5x^3 - 5x^2 + 10x - 1) \div (x - \frac{1}{3})$

40. $(4x^4 - 11x^3 + 18x^2 - 5x + 4) \div (x - \frac{3}{4})$

41. $(5x^4 - 4x^3 + 2x - 5) \div (x - 0.2)$

42. $(3x^4 - 4x^2 + 5x + 8) \div (x + 0.8)$

43. $(5x^5 + 2x^4 + 4x^3 + 6x^2 - 6) \div (x + 0.6)$

44. $(10x^5 - 4x^4 + 2x^2 + 4x - 1) \div (x - 0.4)$

In Problems 45–52, graph each polynomial function using synthetic division and the remainder theorem. Then describe each graph verbally, including the number of x intercepts, the number of turning points, and the left and right behavior.

**Check your work in Problems 45–52 by graphing on a graphing utility.*

45. $P(x) = x^3 - 5x^2 + 2x + 8, -2 \le x \le 5$

46. $P(x) = x^3 + 2x^2 - 5x - 6, -4 \le x \le 3$

47. $P(x) = x^3 + 4x^2 - x - 4, -5 \le x \le 2$

48. $P(x) = x^3 - 2x^2 - 5x + 6, -3 \le x \le 4$

49. $P(x) = -x^3 + 2x^2 - 3, -2 \le x \le 3$

*Please note that use of a graphing utility is not required to complete these exercises. Checking them with a g.u. is optional.

19. $3x^3 - 5x^2 - 4x + 6; x + 1$

20. $3x^3 - 5x^2 - 4x + 6; x - 1$

B

For each polynomial in Problems 21–26, list all possible rational zeros (Theorem 6).

21. $P(x) = x^3 - 3x^2 + 2x - 10$

22. $P(x) = x^3 + 5x^2 - 8x + 14$

23. $P(x) = 2x^3 + 9x^2 - 6x + 5$

24. $P(x) = 3x^3 - 2x^2 - 4x + 2$

25. $P(x) = 6x^3 + 5x^2 + 2x - 25$

26. $P(x) = 10x^3 + 2x^2 - 7x - 9$

In Problems 27–32, write P(x) as a product of linear terms.

27. $P(x) = x^3 + 9x^2 + 24x + 16$; -1 is a zero

28. $P(x) = x^3 - 4x^2 - 3x + 18$; 3 is a double zero

29. $P(x) = x^4 - 1$; 1 and -1 are zeros

30. $P(x) = x^4 + 2x^2 + 1$; i is a double zero

31. $P(x) = 2x^3 - 17x^2 + 90x - 41$; $\frac{1}{2}$ is a zero

32. $P(x) = 3x^3 - 10x^2 + 31x + 26$; $-\frac{2}{3}$ is a zero

In Problems 33–40, find all roots exactly (rational, irrational, and imaginary) for each polynomial equation.

33. $2x^3 - 7x^2 + 2x + 6 = 0$

34. $2x^3 - 7x^2 - 6x - 1 = 0$

35. $x^4 + 2x^3 - 2x^2 - 6x - 3 = 0$

36. $x^4 - 11x^2 + 12x + 4 = 0$

37. $x^4 + 2x^3 - 10x^2 - 18x + 9 = 0$

38. $x^4 - 2x^3 + 9x^2 + 2x - 10 = 0$

39. $2x^5 - x^4 - 5x^3 + 10x^2 - 2x - 4 = 0$

40. $3x^5 + 10x^4 + 4x^3 - 20x^2 - 7x + 10 = 0$

In Problems 41–48, find all zeros exactly (rational, irrational, and imaginary) for each polynomial.

41. $P(x) = x^3 + 5x^2 - 2x - 24$

42. $P(x) = x^3 - 4x^2 - 9x + 36$

43. $P(x) = x^4 - 3.3x^3 + 2.3x^2 + 0.6x$

44. $P(x) = x^4 - 4.1x^3 + 0.1x^2 + 1.2x$

45. $P(x) = x^4 - 2x^3 - 14x^2 + 30x + 9$

46. $P(x) = x^4 + 9x^3 + 23x^2 + 8x - 16$

47. $P(x) = 3x^5 - 2x^4 + 6x^3 + 20x^2 - x - 10$

48. $P(x) = 4x^5 - 18x^4 + 24x^3 - 7x^2 - 4x + 4$

In Problems 49–54, write each polynomial as a product of linear factors.

49. $P(x) = 6x^3 + 19x^2 + 11x - 6$

50. $P(x) = 6x^3 - 11x^2 - 4x + 4$

51. $P(x) = x^3 - x^2 - 13x - 3$

52. $P(x) = x^3 - 4x^2 + 2x + 4$

53. $P(x) = 4x^4 - 4x^3 - 19x^2 + 16x + 12$

54. $P(x) = 4x^4 + 16x^3 + 7x^2 - 18x - 9$

In Problems 55–60, solve each inequality (see Section 2-8).

55. $x^2 \le 4x - 1$

56. $x^2 > 2x + 1$

57. $x^3 + 3 \le 3x^2 + x$

58. $9x + 9 \le x^3 + x^2$

59. $2x^3 + 6 \ge 13x - x^2$

60. $5x^3 - 3x^2 < 10x - 6$

In Problems 61–64, multiply.

61. $[x - (4 - 5i)][x - (4 + 5i)]$

62. $[x - (5 + 2i)][x - (5 - 2i)]$

63. $[x - (a + bi)][x - (a - bi)]$

64. $(x - bi)(x + bi)$

C

In Problems 65–70, find all other zeros of P(x), given the indicated zero.

65. $P(x) = x^3 + x + 10$; $1 + 2i$ is one zero

66. $P(x) = x^3 + 2x^2 - 3x - 10$, $-2 + i$ is one zero

67. $P(x) = x^3 + 4x^2 + 9x + 36$; $-3i$ is one zero

68. $P(x) = x^3 - 5x^2 + 4x - 20$; $2i$ is one zero

69. $P(x) = x^4 - 8x^3 + 24x^2 - 20x - 13$; $3 - 2i$ is one zero

70. $P(x) = x^4 - 6x^3 + 19x^2 - 42x + 10$; $1 - 3i$ is one zero

In Problems 71–74, solve each inequality (see Section 2-8).

71. $\dfrac{4}{2x^3 + 5x^2 - 2x - 5} \ge 0$

72. $\dfrac{7}{2x^3 - x^2 - 8x + 4} \le 0$

73. $\dfrac{x^2 - 3x - 10}{x^3 - 4x^2 + x + 6} \le 0$

74. $\dfrac{x^2 + 4x - 21}{x^3 + 7x^2 + 7x - 15} \ge 0$

Problems 75–80 require the use of a graphing utility. Graph the polynomial and use the graph to help locate the real zeros. Then find all zeros (rational, irrational, and imaginary) exactly.

75. $P(x) = 3x^3 - 37x^2 + 84x - 24$

76. $P(x) = 2x^3 - 9x^2 - 2x + 30$

77. $P(x) = 4x^4 + 4x^3 + 49x^2 + 64x - 240$

78. $P(x) = 6x^4 + 35x^3 + 2x^2 - 233x - 360$

79. $P(x) = 4x^4 - 44x^3 + 145x^2 - 192x + 90$

80. $P(x) = x^5 - 6x^4 + 6x^3 + 28x^2 - 72x + 48$

81. The solutions to the equation $x^3 - 1 = 0$ are all the cube roots of 1.
(A) How many cube roots of 1 are there?
(B) 1 is obviously a cube root of 1; find all others.

82. The solutions to the equation $x^3 - 8 = 0$ are all the cube roots of 8.
(A) How many cube roots of 8 are there?
(B) 2 is obviously a cube root of 8; find all others.

83. If P is a polynomial function with real coefficients of degree n, with n odd, then what is the maximum number of times the graph of $y = P(x)$ can cross the x axis? What is the minimum number of times?

84. Answer the questions in Problem 83 for n even.

85. Given $P(x) = x^2 + 2ix - 5$ with $2 - i$ a zero, show that $2 + i$ is not a zero of $P(x)$. Does this contradict Theorem 4? Explain.

86. If $P(x)$ and $Q(x)$ are two polynomials of degree n, and if $P(x) = Q(x)$ for more than n values of x, then how are $P(x)$ and $Q(x)$ related?

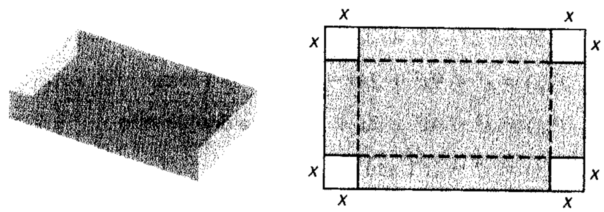
APPLICATIONS

Find all rational solutions exactly, and find irrational solutions to two decimal places.

87. **Storage.** A rectangular storage unit has dimensions 1 by 2 by 3 feet. If each dimension is increased by the same amount, how much should this amount be to create a new storage unit with volume ten times the old?

88. **Construction.** A rectangular box has dimensions 1 by 1 by 2 feet. If each dimension is increased by the same amount, how much should this amount be to create a new box with volume six times the old?

★ 89. **Packaging.** An open box is to be made from a rectangular piece of cardboard that measures 8 by 5 inches, by cutting out squares of the same size from each corner and bending up the sides (see the figure). If the volume of the box is to be 14 cubic inches, how large a square should be cut from each corner? [*Hint:* Determine the domain of x from physical considerations before starting.]



★ 90. **Fabrication.** An open metal chemical tank is to be made from a rectangular piece of stainless steel that measures 10 by 8 feet, by cutting out squares of the same size from each corner and bending up the sides (see the figure). If the volume of the tank is to be 48 cubic feet, how large a square should be cut from each corner?

SECTION 4-3 Approximating Real Zeros of Polynomials

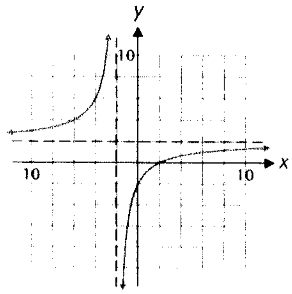
- Locating Real Zeros
- The Bisection Method
- Approximating Real Zeros Using a Graphing Utility
- Application

The strategy for finding zeros discussed in the preceding section is designed to find as many exact real and imaginary zeros as possible. But there are zeros that cannot be found by using the strategy. For example, the polynomial

$$P(x) = x^5 + x - 1$$

must have at least one real zero (Theorem 5 in Section 4-2). Since the only possible rational zeros are ± 1 and neither of these turns out to be a zero, $P(x)$ must have at least one irrational zero. We cannot find the exact value of this zero, but it can be approximated using various well-known methods.

4.



In Problems 5–12, find the domain and x intercepts. Do not graph.

5. $f(x) = \dfrac{2x-4}{x+1}$

6. $g(x) = \dfrac{3x+6}{x-1}$

7. $h(x) = \dfrac{x^2-1}{x^2-16}$

8. $k(x) = \dfrac{x^2-36}{x^2-25}$

9. $r(x) = \dfrac{x^2-x-6}{x^2-x-12}$

10. $s(x) = \dfrac{x^2+x-12}{x^2+x-6}$

11. $F(x) = \dfrac{x}{x^2+4}$

12. $G(x) = \dfrac{x^2}{x^2+16}$

In Problems 13–20, find all vertical and horizontal asymptotes. Do not graph.

13. $f(x) = \dfrac{2x}{x-4}$

14. $h(x) = \dfrac{3x}{x+5}$

15. $s(x) = \dfrac{2x^2+3x}{3x^2-48}$

16. $r(x) = \dfrac{5x^2-7x}{2x^2-50}$

17. $p(x) = \dfrac{2x}{x^4+1}$

18. $q(x) = \dfrac{5x^4}{2x^2+3x-2}$

19. $t(x) = \dfrac{6x^4}{3x^2-2x-5}$

20. $g(x) = \dfrac{3x}{x^4+2x^2+1}$

B

In Problems 21–40, use the graphing strategy outlined in the text to sketch the graph of each function.

Check Problems 21–40 on a graphing utility.

21. $f(x) = \dfrac{1}{x-4}$

22. $g(x) = \dfrac{1}{x+3}$

23. $f(x) = \dfrac{x}{x+1}$

24. $f(x) = \dfrac{3x}{x-3}$

25. $h(x) = \dfrac{x}{2x-2}$

26. $p(x) = \dfrac{3x}{4x+4}$

27. $f(x) = \dfrac{2x-4}{x+3}$

28. $f(x) = \dfrac{3x+3}{2-x}$

29. $g(x) = \dfrac{1-x^2}{x^2}$

30. $f(x) = \dfrac{x^2+1}{x^2}$

31. $f(x) = \dfrac{9}{x^2-9}$

32. $g(x) = \dfrac{6}{x^2-x-6}$

33. $f(x) = \dfrac{x}{x^2-1}$

34. $p(x) = \dfrac{x}{1-x^2}$

35. $g(x) = \dfrac{2}{x^2+1}$

36. $f(x) = \dfrac{x}{x^2+1}$

37. $f(x) = \dfrac{12x^2}{(3x+5)^2}$

38. $f(x) = \dfrac{7x^2}{(2x-3)^2}$

39. $f(x) = \dfrac{x^2-1}{x^2+7x+10}$

40. $f(x) = \dfrac{x^2+6x+8}{x^2-x-2}$

41. If $f(x) = n(x)/d(x)$, where $n(x)$ and $d(x)$ are quadratic functions, what is the maximum number of x intercepts $f(x)$ can have? What is the minimum number? Illustrate both cases with examples.

42. If $f(x) = n(x)/d(x)$, where $n(x)$ and $d(x)$ are quadratic functions, what is the maximum number of vertical asymptotes $f(x)$ can have? What is the minimum number? Illustrate both cases with examples.

In Problems 43–48, find all vertical, horizontal, and oblique asymptotes. Do not graph.

43. $f(x) = \dfrac{2x^2}{x-1}$

44. $g(x) = \dfrac{3x^2}{x+2}$

45. $p(x) = \dfrac{x^3}{x^2+1}$

46. $q(x) = \dfrac{x^5}{x^3-8}$

47. $r(x) = \dfrac{2x^2-3x+5}{x}$

48. $s(x) = \dfrac{-3x^2+5x+9}{x}$

In Problems 49–52, use a graphing utility to investigate the behavior of each function as $x \to \infty$ and as $x \to -\infty$, and to find any horizontal asymptotes.

49. $f(x) = \dfrac{5x}{\sqrt{x^2+1}}$

50. $f(x) = \dfrac{2x}{\sqrt{x^2-1}}$

51. $f(x) = \dfrac{4\sqrt{x^2-4}}{x}$

52. $f(x) = \dfrac{3\sqrt{x^2+1}}{x-1}$

C

In Problems 53–58, use the graphing strategy outlined in the text to sketch the graph of each function. Include any oblique asymptotes.

Check Problems 53–58 with a graphing utility.

53. $f(x) = \dfrac{x^2+1}{x}$

54. $g(x) = \dfrac{x^2-1}{x}$

EXERCISE 5-1

A

In Problems 1–10, construct a table of values for integer values of x over the indicated interval and then graph the function.

1. $y = 3^x$; $[-3, 3]$
2. $y = 5^x$; $[-2, 2]$
3. $y = (\frac{1}{3})^x = 3^{-x}$; $[-3, 3]$
4. $y = (\frac{1}{5})^x = 5^{-x}$; $[-2, 2]$
5. $g(x) = -3^{-x}$; $[-3, 3]$
6. $f(x) = -5^x$; $[-2, 2]$
7. $h(x) = 5(3^x)$; $[-3, 3]$
8. $f(x) = 4(5^x)$; $[-2, 2]$
9. $y = 3^{x+3} - 5$; $[-6, 0]$
10. $y = 5^{x+2} + 4$; $[-4, 0]$

In Problems 11–22, simplify.

11. $2^{5x+1}2^{3-2x}$
12. $5^{2x-3}5^{6-4x}$
13. $\dfrac{3^{2x+x}}{3^{3x+5x}}$
14. $\dfrac{7^{3x-x}}{7^{4x-6x}}$
15. $(4^x)^3$
16. $(x^3)^4$
17. $(10^2)^x(10^x)^2$
18. 2^x2^x
19. $\left(\dfrac{5^x}{4^x}\right)^3$
20. $\left(\dfrac{3^{2x}}{7^{3x}}\right)^2$
21. $\left(\dfrac{a^{-2}b^3c^{-1}}{a^{-3}b^2c^{-3}}\right)^2$
22. $\left(\dfrac{a^{-4}b^3c^2}{a^{-6}b^2c^{-1}}\right)^3$

B

In Problems 23–34, solve for x.

23. $3^{2x-5} = 3^{4x+2}$
24. $4^{3x+1} = 4^{2x-2}$
25. $10^{x^2+2} = 10^{2x+2}$
26. $5^{x^2-5} = 5^{3x+5}$
27. $(2x + 1)^3 = 8$
28. $(2x - 1)^5 = -32$
29. $5^{3x} = 25^{x+3}$
30. $4^{5x-1} = 16^{2x-1}$
31. $4^{2x+2} = 8^{x+2}$
32. $100^{2x+4} = 1{,}000^{x+4}$
33. $100^{x^2} = 10^{5x-3}$
34. $3^{x^2} = 9^{x+4}$

35. Find all real numbers a such that $a^2 = a^{-2}$. Explain why this does not violate the second exponential function property in the box on page 358.

36. Find real numbers a and b such that $a \neq b$ but $a^4 = b^4$. Explain why this does not violate the third exponential function property in the box on page 358.

Graph each function in Problems 37–46 by constructing a table of values.

**Check Problems 37–46 with a graphing utility.*

37. $G(t) = 3^{t/100}$
38. $f(t) = 2^{t/10}$
39. $y = 11(3^{-x/2})$
40. $y = 7(2^{-2x})$
41. $g(x) = 2^{-|x|}$
42. $f(x) = 2^{|x|}$
43. $y = 1{,}000(1.08)^x$
44. $y = 100(1.03)^x$
45. $y = 2^{-x^2}$
46. $y = 3^{-x^2}$

*Please note that use of graphing utility is not required to complete these exercises. Checking them with a g.u. is optional.

C

In Problems 47–50, simplify.

47. $(6^x + 6^{-x})(6^x - 6^{-x})$
48. $(3^x - 3^{-x})(3^x + 3^{-x})$
49. $(6^x + 6^{-x})^2 - (6^x - 6^{-x})^2$
50. $(3^x - 3^{-x})^2 + (3^x + 3^{-x})^2$

Graph each function in Problems 51–54 by constructing a table of values.

Check Problems 51–54 with a graphing utility.

51. $m(x) = x(3^{-x})$
52. $h(x) = x(2^x)$
53. $f(x) = \dfrac{2^x + 2^{-x}}{2}$
54. $g(x) = \dfrac{3^x + 3^{-x}}{2}$

In Problems 55–58:

(A) Approximate the real zeros of each function to two decimal places.

(B) Investigate the behavior of each function as $x \to \infty$ and $x \to -\infty$ and find any horizontal asymptotes.

55. $f(x) = 3^x - 5$
56. $f(x) = 4 + 2^{-x}$
57. $f(x) = 1 + x + 10^x$
58. $f(x) = 8 - x^2 + 2^{-x}$

APPLICATIONS

59. **Gaming.** A person bets on red and black on a roulette wheel using a *Martingale strategy*. That is, a $2 bet is placed on red, and the bet is doubled each time until a win occurs. The process is then repeated. If black occurs n times in a row, then $L = 2^n$ dollars is lost on the nth bet. Graph this function for $1 \leq n \leq 10$. Even though the function is defined only for positive integers, points on this type of graph are usually jointed with a smooth curve as a visual aid.

60. **Bacterial Growth.** If bacteria in a certain culture double every $\frac{1}{2}$ hour, write an equation that gives the number of bacteria N in the culture after t hours, assuming the culture has 100 bacteria at the start. Graph the equation for $0 \leq t \leq 5$.

61. **Population Growth.** Because of its short life span and frequent breeding, the fruit fly *Drosophila* is used in some genetic studies. Raymond Pearl of Johns Hopkins University,