A major manufacturing company is planning to optimise its international supply chain. The company currently produces three major lines of products at three plants located in three different countries. Its customers are distributed all over the world but can be grouped into five major geographical areas. The unit costs of transportation, which depend on the origin, destination and product type, are in terms of S$ per kg, and are mainly accrued in the sea transportation links. The supply and demand points to the supply chain are all located near ports, therefore, the land freight charges can be omitted.

Formulate appropriate mathematical programming models for each of the following scenarios.

(a) Base case (direct shipment) scenario:
The products are shipped directly from each of the three plants to customers in each of the five geographical areas. What are the requirements for the formulated model to have feasible solutions? What are the requirements if integer solutions are desired?

(b) Mixed hub and direct shipment scenario:
The company is planning to set up an international distribution hub to help consolidate the shipments. The unit costs of transportation using less-than-container load are known and depend on the origin, destination and the product type. As a general rule of thumb, the full container load (FCL) shipping costs are about 60% of the less-than-container load (LCL) shipping costs. For simplicity, you can assume that the company can achieve shipping out 60% of the products from the hub using full container load. The setup cost for one international hub is $100,000, while the operating cost at the hub is $0.50 per kg of inbound and $1.20 per kg of outbound products.

(c) Discuss how the optimal solution of the mixed-binary integer programming problem set up in part (b) can be obtained and comment on the four possible outcomes of the optimal solution.