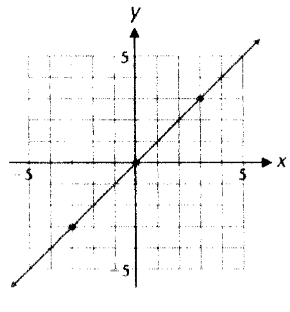
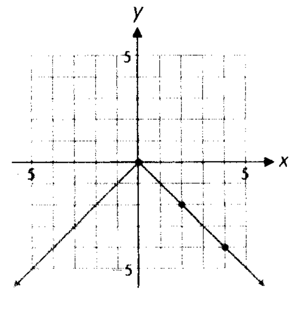
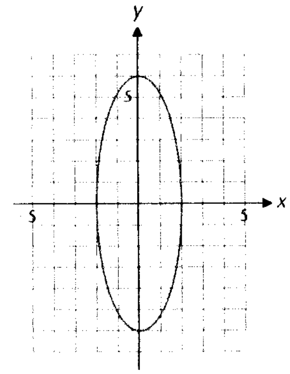
3. (A) Symmetric with respect to the origin



(B) Symmetric with respect to the y axis

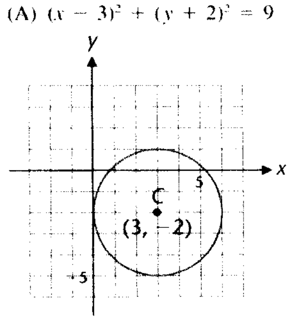


(C) Symmetric with respect to the x axis, y axis, and origin

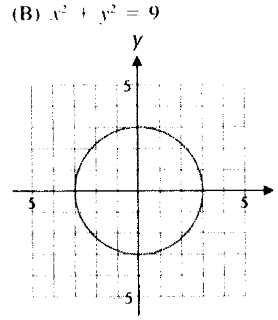


4. $d = \sqrt{173}$

5. (A) $(x - 3)^2 + (y + 2)^2 = 9$



(B) $x^2 + y^2 = 9$



6. $(x - 4)^2 + (y + 5)^2 = 16$; radius: 4, center: $(4, -5)$

EXERCISE 3-1

A

In Problems 1–10, give a verbal description of the indicated subset of the plane in terms of quadrants and axes.

1. $\{(x, y) \mid x = 0\}$
2. $\{(x, y) \mid x > 0, y > 0\}$
3. $\{(x, y) \mid x < 0, y < 0\}$
4. $\{(x, y) \mid y = 0\}$
5. $\{(x, y) \mid x > 0, y < 0\}$
6. $\{(x, y) \mid y < 0, x \neq 0\}$
7. $\{(x, y) \mid xy < 0\}$
8. $\{(x, y) \mid x < 0, y > 0\}$
9. $\{(x, y) \mid x > 0, y \neq 0\}$
10. $\{(x, y) \mid xy > 0\}$

In Problems 11–18, determine symmetry with respect to the x axis, y axis, or origin, if any exists, and graph.

11. $y = 2x - 4$
12. $y = \frac{1}{2}x + 1$
13. $y = \frac{1}{2}x$
14. $y = 2x$
15. $|y| = x$
16. $|y| = -x$
17. $|x| = |y|$
18. $y = -x$

Find the distance between the indicated points in Problems 19–22. Leave the answer in radical form.

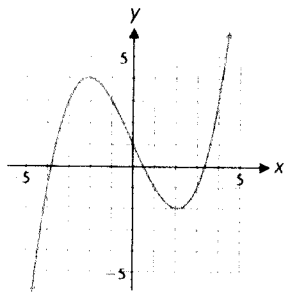
19. $(-5, -3), (4, 2)$
20. $(-6, 4), (2, -1)$
21. $(3, 5), (2, -4)$
22. $(2, -5), (-3, 1)$

In Problems 23–28, write the equation of a circle with the indicated center and radius.

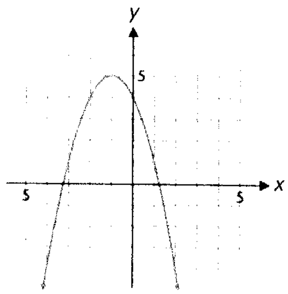
23. $C(0, 0), r = 4$
24. $C(0, 0), r = 6$
25. $C(3, -2), r = 1$
26. $C(-4, 2), r = 5$
27. $C(2, 6), r = \sqrt{3}$
28. $C(-1, -3), r = \sqrt{5}$

In Problems 29 and 30, use the graph to estimate to the nearest integer the missing coordinates of the indicated points. (Be sure you find all possible answers.)

29. (A) (−3, ?)
(B) (2, ?)
(C) (?, 3)
(D) (?, −1)

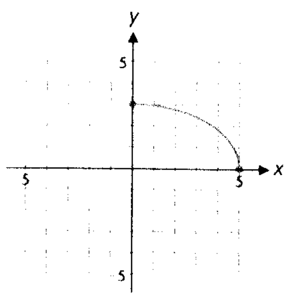


30. (A) (−4, ?)
(B) (−1, ?)
(C) (?, 1)
(D) (?, 4)

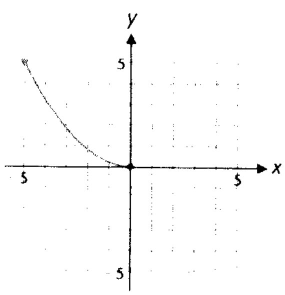


The figures in Problems 31 and 32 show a portion of a graph. Extend the given graph to one that exhibits the indicated type of symmetry.

31. (A) x axis only
(B) y axis only
(C) origin only
(D) x and y axes



32. (A) x axis only
(B) y axis only
(C) origin only
(D) x and y axes



B

In Problems 33–48, determine symmetry with respect to the x axis, y axis, or origin, if any exists, and graph.

* *Check your graphs in Problems 33–48 by graphing on a graphing utility.*

33. $y^2 = x + 2$ **34.** $y^2 = x - 2$ **35.** $y = x^2 + 1$

36. $y + 2 = x^2$ **37.** $x^2 + 4y^2 = 4$ **38.** $x^2 + 9y^2 = 9$

39. $4y^2 - x^2 = 1$ **40.** $4x^2 - y^2 = 1$ **41.** $y^3 = x$

42. $y = x^4$ **43.** $y = 0.6x^2 - 4.5$

44. $x = 0.8y^2 - 3.5$ **45.** $y = \sqrt{17 - x^2}$

46. $y = \sqrt{100 - 4x^2}$ **47.** $y = x^{2/3}$

48. $y^{2/3} = x$

In Problems 49 and 50, use the Pythagorean theorem to show that the given points are the vertices of a right triangle. Find the area and the perimeter (to two decimal places) of the triangle. (Formulas related to triangles can be found in Appendix B.)

49. $(-3, 2), (1, -2), (8, 5)$

50. $(-4, -1), (0, 7), (6, -6)$

51. Find x such that $(x, 7)$ is 10 units from $(-4, 1)$.

52. Find x such that $(x, 2)$ is 4 units from $(3, -3)$.

53. Find y such that $(2, y)$ is 3 units from $(-1, 4)$.

54. Find y such that $(3, y)$ is 13 units from $(-9, 2)$.

In Problems 55–60, find the center and radius of the circle with the given equation. Graph the equation.

55. $(x + 4)^2 + (y - 2)^2 = 7$

56. $(x - 5)^2 + (y + 7)^2 = 15$

57. $x^2 + y^2 - 6x - 4y = 36$

58. $x^2 + y^2 - 2x - 10y = 55$

59. $x^2 + y^2 + 8x - 6y + 8 = 0$

60. $x^2 + y^2 + 4x + 10y + 15 = 0$

In Problems 61–64, graph the triangle with vertices A, B, and C and the triangle with vertices A′, B′, and C′ in the same coordinate system. Describe the relationship between the graphs of these triangles in terms of reflections.

61. $A(1, 1), B(7, 2), C(4, 6)$
$A'(1, -1), B'(7, -2), C'(4, -6)$

62. $A(1, 1), B(7, 2), C(4, 6)$
$A'(-1, 1), B'(-7, 2), C'(-4, 6)$

63. $A(1, 1), B(7, 2), C(4, 6)$
$A'(-1, -1), B'(-7, -2), C'(-4, -6)$

64. $A(1, 2), B(1, 4), C(3, 4)$
$A'(2, 1), B'(4, 1), C'(4, 3)$
Hint: Add the graph of $y = x$ to your graph.

In Problems 65–68, solve for y, producing two equations, and then graph both of these equations in the same viewing window.

65. $x^2 + y^2 = 3$ **66.** $x^2 + y^2 = 5$

67. $(x + 3)^2 + (y + 1)^2 = 2$ **68.** $(x - 2)^2 + (y - 1)^2 = 3$

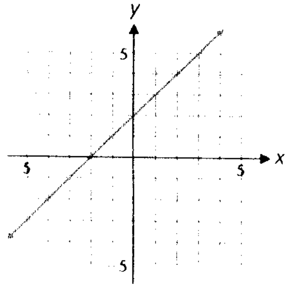
*Please note that use of a graphing utility is not required to complete these exercises. Checking with a g.u. is optional. If you do not have a g.u., you should still work these exercises.

EXERCISE 3-2

A

In Problems 1–6, use the graph of each line to find the x intercept, y intercept, and slope. Write the slope-intercept form of the equation of the line.

1.



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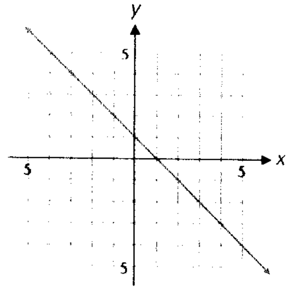
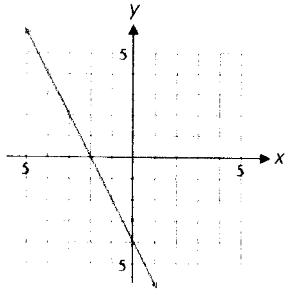


3.



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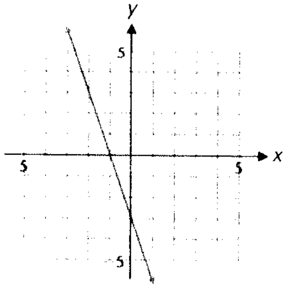
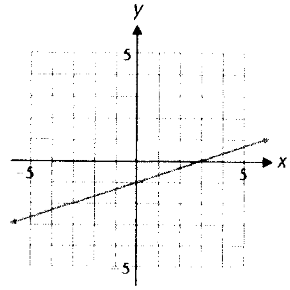
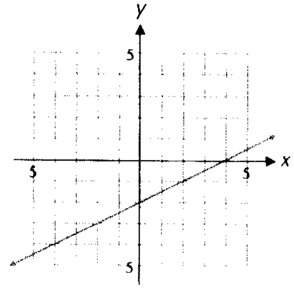


5.



6.



Graph each equation in Problems 7–20, and indicate the slope, if it exists.

Check your graphs in Problems 7–20 by graphing each on a graphing utility.

7. $y = -\frac{3}{5}x + 4$ **8.** $y = -\frac{3}{2}x + 6$

9. $y = -\frac{3}{4}x$ **10.** $y = \frac{2}{3}x - 3$

11. $2x - 3y = 15$ **12.** $4x + 3y = 24$

13. $4x - 5y = -24$ **14.** $6x - 7y = -49$

15. $\frac{y}{8} - \frac{x}{4} = 1$ **16.** $\frac{y}{6} - \frac{x}{5} = 1$

17. $x = -3$ **18.** $y = -2$

19. $y = 3.5$ **20.** $x = 2.5$

In Problems 21–24, find the equation of the line with the indicated slope and y intercept. Write the final answer in the standard form $Ax + By = C$, $A \geq 0$.

21. Slope = 1; y intercept = 0

22. Slope = −1; y intercept = 7

23. Slope = $-\frac{2}{3}$; y intercept = −4

24. Slope = $\frac{5}{3}$; y intercept = 6

B

In Problems 25–28, find the equation of the line passing through the given point with the given slope. Write the final answer in the slope-intercept form $y = mx + b$.

25. $(0, 3)$; $m = -2$

26. $(4, 0)$; $m = 3$

27. $(-5, 4)$; $m = \frac{3}{2}$

28. $(2, -3)$; $m = -\frac{4}{5}$

In Problems 29–34, find the equation of the line passing through the two given points. Write the final answer in the slope-intercept form $y = mx + b$ or in the form $x = c$.

29. $(2, 5)$; $(4, -3)$

30. $(-1, 4)$; $(3, 2)$

31. $(-3, 2)$; $(-3, 5)$

32. $(0, 5)$; $(2, 5)$

33. $(-4, 2)$; $(0, 2)$

34. $(5, -4)$; $(5, 6)$

In Problems 35–46, write an equation of the line that contains the indicated point and meets the indicated condition(s). Write the final answer in the standard form $Ax + By = C$, $A \geq 0$.

35. $(2, -1)$; parallel to $y = -3x + 7$

36. $(-3, 2)$; parallel to $y = 4x - 5$

37. $(0, -4)$; parallel to $2x + 3y = 9$

38. $(-2, 0)$; parallel to $-3x + 4y = 10$

39. $(3, 3)$; parallel to x axis

40. $(-2, -1)$; parallel to y axis

41. $(4, 5)$; perpendicular to $y = \frac{3}{2}x - 4$

42. $(-1, 3)$; perpendicular to $y = -\frac{3}{5}x + 2$

43. $(5, 0)$; perpendicular to $-5x + 2y = 1$

44. $(0, 3)$; perpendicular to $2x + y = 1$

45. $(-2, -3)$; perpendicular to x axis

46. $(1, -7)$; perpendicular to y axis

In Problems 47–50, classify the quadrilateral ABCD with the indicated vertices as a trapezoid, a parallelogram, a rectangle, or none of these.

47. $A(-2, 2)$; $B(8, 7)$; $C(10, 1)$; $D(-4, -6)$

48. $A(-5, -2)$; $B(-3, 4)$; $C(6, 10)$; $D(4, 4)$

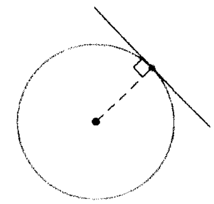
49. $A(0, 2)$; $B(4, -1)$; $C(1, -5)$; $D(-3, -2)$

50. $A(-6, 3)$; $B(3, 7)$; $C(2, 4)$; $D(-4, -1)$

51. Find the equation of the perpendicular bisector of the line segment joining $(-4, -3)$ and $(2, 4)$ by using the point-slope form of the equation of a line.

52. Solve Problem 51 by using the distance between two points formula, and compare the results.

Problems 53–58 are calculus-related. Recall that a line tangent to a circle at a point is perpendicular to the radius drawn to that point (see the figure). Find the equation of the line tangent to the circle at the indicated point. Write the final answer in the standard form $Ax + By = C$, $A \geq 0$. Graph the circle and the tangent line on the same coordinate system.



53. $x^2 + y^2 = 25$, $(3, 4)$

54. $x^2 + y^2 = 100$, $(-8, 6)$

55. $x^2 + y^2 = 50$, $(5, -5)$

56. $x^2 + y^2 = 80$, $(-4, -8)$

57. $(x - 3)^2 + (y + 4)^2 = 169$, $(8, -16)$

58. $(x + 5)^2 + (y - 9)^2 = 289$, $(-13, -6)$

C

59. (A) Graph the following equations in the same coordinate system:

$$3x + 2y = 6 \qquad 3x + 2y = 3$$

$$3x + 2y = -6 \qquad 3x + 2y = -3$$

(B) From your observations in part A, describe the family of lines obtained by varying C in $Ax + By = C$ while holding A and B fixed.

(C) Verify your conclusions in part B with a proof.

60. (A) Graph the following two equations in the same coordinate system:

$$3x + 4y = 12 \qquad 4x - 3y = 12$$

(B) Graph the following two equations in the same coordinate system:

$$2x + 3y = 12 \qquad 3x - 2y = 12$$

(C) From your observations in parts A and B, describe the apparent relationship of the graphs of $Ax + By = C$ and $Bx - Ay = C$.

(D) Verify your conclusions in part C with a proof.

The form of the definition of function that has been used until well into the twentieth century (many texts still contain this definition) was formulated by Dirichlet (1805–1859). He stated that, if two variables x and y are so related that for each value of x there corresponds exactly one value of y, then y is said to be a (single-valued) function of x. He called x, the variable to which values are assigned at will, the independent variable, and y, the variable whose values depend on the values assigned to x, the dependent variable. He called the values assumed by x the domain of the function, and the corresponding values assumed by y the range of the function.

Now, since set concepts permeate almost all mathematics, we have the more general definition of function presented in this section in terms of sets of ordered pairs of elements.

Answers to Matched Problems

1. (A) S does not define a function
(B) T defines a function with domain $\{-2, -1, 0, 1, 2\}$ and range $\{0, 1, 2\}$
2. (A) Does not define a function (B) Defines a function
3. $x \geq -5$ Inequality notation
$[-5, \infty)$ Interval notation
4. (A) -3 (B) -2 (C) Does not exist (D) 1
5. Domain of F: all real numbers
Domain of G: $x < -3$ or $x \geq 2$ Inequality notation
$(-\infty, -3) \cup [2, \infty)$ Interval notation
Domain of H: All real numbers except 2
6. (A) $x^2 + 2xh + h^2 + 3x + 3h + 7$ (B) $2x + h + 3$
7. (A) $A(x) = x(100 - 2x)$ (B) Domain: $0 < x < 50$ Inequality notation
$(0, 50)$ Interval notation

EXERCISE 3-3

A

Indicate whether each table in Problems 1–6 defines a function.

1.

Domain	Range
−1	1
0	2
1	3

2.

Domain	Range
2	1
4	3
6	5

3.

Domain	Range
1	3
3	5
	7
5	9

4.

Domain	Range
−1	0
−2	5
−3	8

5.

Domain	Range
−1	
0	3
1	
2	

6.

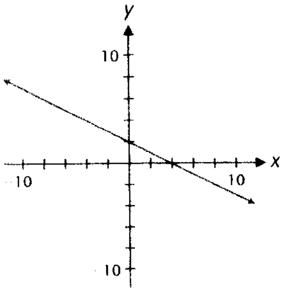
Domain	Range
2	8
3	
4	9
5	

Indicate whether each set in Problems 7–12 defines a function. Find the domain and range of each function.

7. $\{(2, 4), (3, 6), (4, 8), (5, 10)\}$
8. $\{(-1, 4), (0, 3), (1, 2), (2, 1)\}$
9. $\{(10, -10), (5, -5), (0, 0), (5, 5), (10, 10)\}$
10. $\{(-10, 10), (-5, 5), (0, 0), (5, 5), (10, 10)\}$
11. $\{(0, 1), (1, 1), (2, 1), (3, 2), (4, 2), (5, 2)\}$
12. $\{(1, 1), (2, 1), (3, 1), (1, 2), (2, 2), (3, 2)\}$

Indicate whether each graph in Problems 13–18 is the graph of a function.

13.



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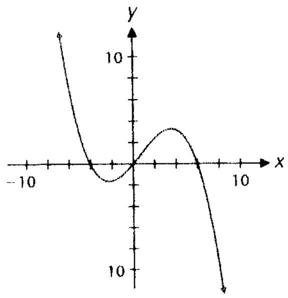
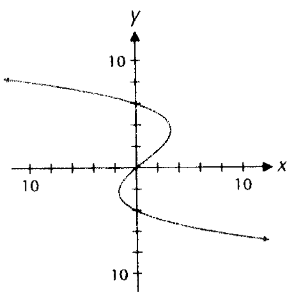


15.



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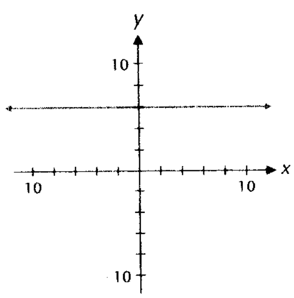
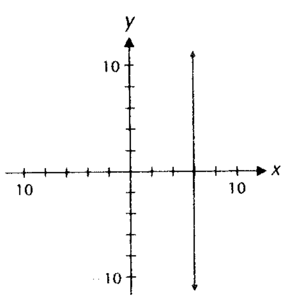
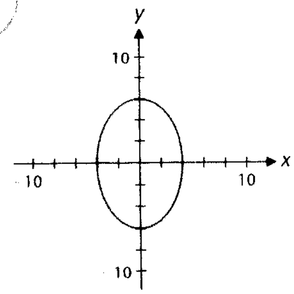


17.



18.



Problems 19–28 refer to the functions

$$f(x) = 2x + 6 \qquad g(t) = 3 - 2t$$
$$F(m) = 2m^2 + 3m - 1 \qquad G(u) = u^2 + u - 2$$

Evaluate as indicated.

19. $f(-1)$

20. $g(6)$

21. $G(-2)$

22. $F(-3)$

23. $F(-1) + f(3)$

24. $G(2) - g(-3)$

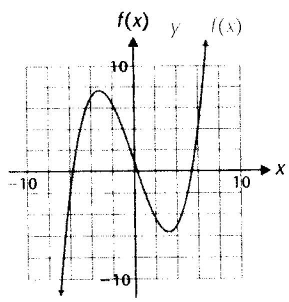
25. $2F(-2) - G(-1)$

26. $3G(-2) + 2F(-1)$

27. $\dfrac{f(0) \cdot g(-2)}{F(-3)}$

28. $\dfrac{g(4) \cdot f(2)}{G(1)}$

In Problems 29–32, use the following graph of a function f to determine x or y to the nearest integer, as indicated. Some problems may have more than one answer.



29. $y = f(-4)$

30. $y = f(4)$

31. $4 = f(x)$

32. $-2 = f(x)$

B

Determine which of the equations in Problems 33–42 define a function with independent variable x. For those that do, find the domain. For those that do not, find a value of x to which there corresponds more than one value of y.

33. $y - x^2 = 1$

34. $y^2 - x = 1$

35. $2x^3 + y^2 = 4$

36. $3x^2 + y^3 = 8$

37. $x^3 - y = 2$ **38.** $x^3 + |y| = 6$

39. $xy = 0$ **40.** $xy = 1$

41. $x^2 + xy = 1$ **42.** $x^2 + xy = 0$

In Problems 43–56, find the domain of the indicated function.

43. $f(x) = \sqrt{x - 2}$ **44.** $g(t) = \sqrt[3]{5 - t}$

45. $h(u) = \sqrt[3]{u + 9}$ **46.** $k(s) = \sqrt{-1 - s}$

47. $F(w) = \dfrac{2w + 3}{w - 4}$ **48.** $G(m) = \dfrac{1 - 2m}{m^2 + 3}$

49. $H(n) = \dfrac{3n + 7}{n^2 + n - 2}$ **50.** $K(v) = \dfrac{2v^2 - 9}{v^2 - v - 6}$

51. $I(y) = \sqrt{y^2 - 2y - 3}$ **52.** $J(z) = \sqrt{z^2 - 2z + 5}$

53. $f(x) = \sqrt{\dfrac{x^2 - 4x + 5}{x^2 + 2x + 3}}$ **54.** $g(t) = \sqrt{\dfrac{t - 1}{5 - t}}$

55. $F(y) = \sqrt{\dfrac{y - 3}{y + 2}}$ **56.** $G(z) = \sqrt{\dfrac{z^2 + 4z + 6}{z^2 + 4z + 3}}$

The verbal statement "function f multiplies the square of the domain element by 3 and then subtracts 7 from the result" and the algebraic statement "$f(x) = 3x^2 - 7$" define the same function. In Problems 57–60, translate each verbal definition of a function into an algebraic definition.

57. Function g subtracts 5 from twice the cube of the domain element.

58. Function f multiplies the domain element by -3 and adds 4 to the result.

59. Function G multiplies the square root of the domain element by 2 and subtracts the square of the domain element from the result.

60. Function F multiplies the cube of the domain element by -8 and adds three times the square root of 3 to the result.

In Problems 61–64, translate each algebraic definition of the function into a verbal definition.

61. $f(x) = 2x - 3$ 62. $g(x) = -2x + 7$

63. $F(x) = 3x^3 - 2\sqrt{x}$ 64. $G(x) = 4\sqrt{x} - x^2$

65. If $F(s) = 3s + 15$, find: $\dfrac{F(2 + h) - F(2)}{h}$

66. If $K(r) = 7 - 4r$, find: $\dfrac{K(1 + h) - K(1)}{h}$

67. If $g(x) = 2 - x^2$, find: $\dfrac{g(3 + h) - g(3)}{h}$

68. If $P(m) = 2m^2 + 3$, find: $\dfrac{P(2 + h) - P(2)}{h}$

69. If $L(w) = -2w^2 + 3w - 1$, find: $\dfrac{L(-2 + h) - L(-2)}{h}$

70. If $D(p) = -3p^2 - 4p + 9$, find: $\dfrac{D(-1 + h) - D(-1)}{h}$

71. Find $f(x)$, given that

$$f(x + h) = 3(x + h)^2 - 5(x + h) + 9$$

72. Find $g(w)$, given that

$$g(w + h) = -4(w + h)^3 + 7(w + h) - 5$$

73. Find $m(t)$, given that

$$m(t + h) = -2(t + h)^2 - 5\sqrt{t + h} - 2$$

74. Find $s(z)$, given that

$$s(z + h) = 3(z + h) + 9\sqrt{z + h} + 1$$

C

In Problems 75–82, find and simplify:

(A) $\dfrac{f(x + h) - f(x)}{h}$ *(B)* $\dfrac{f(x) - f(a)}{x - a}$

75. $f(x) = 4x - 7$ **76.** $f(x) = -5x + 2$

77. $f(x) = 2x^2 - 4$ **78.** $f(x) = 5 - 3x^2$

79. $f(x) = -4x^2 + 3x - 2$ **80.** $f(x) = 3x^2 - 5x - 9$

81. $f(x) = x^3 - 2x$ **82.** $f(x) = x^2 - x^3$

83. The area of a rectangle is 64 square inches. Express the perimeter $P(w)$ as a function of the width w and state the domain.

84. The perimeter of a rectangle is 50 inches. Express the area $A(w)$ as a function of the width w and state the domain.

85. The altitude of a right triangle is 5 meters. Express the hypotenuse $h(b)$ as a function of the base b and state the domain.

86. The altitude of a right triangle is 4 meters. Express the base $b(h)$ as a function of the hypotenuse h and state the domain.

APPLICATIONS

Most of the applications in this section are calculus-related. That is, similar problems will appear in a calculus course, but additional analysis of the functions will be required.

87. Cost Function. The fixed costs per day for a doughnut shop are \$300, and the variable costs are \$1.75 per dozen doughnuts produced. If x dozen doughnuts are produced daily, express the daily cost $C(x)$ as a function of x.

88. Cost Function. The fixed costs per day for a ski manufacturer are \$3,750, and the variable costs are \$68 per pair of skis produced. If x pairs of skis are produced daily, express the daily cost $C(x)$ as a function of x.