7-13. *(7.9)* A parallel-plate capacitor with a nonuniform dielectric

The dielectric of a parallel-plate capacitor has a permittivity that varies as $\epsilon_{r0} + ax$, where x is the distance from one plate. The area of a plate is $\mathcal{A}$, and their spacing is s.

(a) Find the capacitance.

(b) Show that, if ϵ_r varies from ϵ_{r0} to $2\epsilon_{r0}$, then C is 1.44 times as large as if a were zero.

(c) Find P from the values of D and E for that case.

(d) Deduce the value of ρ_b.

(e) Now calculate ρ_b from the relation given in Prob. 7-2.

(f) Draw curves of E, ρ_b, and P as functions of x for $\epsilon_{r0} = 3.00$, $a = \epsilon_{r0}/s$, $s = 1.00$ millimeter when the applied voltage is 1.00 volt.

This is part of problem 7-2 just for reference for part (e) but it does not need to be solved.

Show that, in a nonhomogeneous dielectric, if $\rho_f = 0$, then $\rho_b = -(\epsilon_0/\epsilon_r)\boldsymbol{E} \cdot \nabla\epsilon_r$.