WINTER 2007

OPERATIONS MODELING

IOE 202

Homework 1

IMPORTANT NOTE This is a team homework. The team works on this homework together, but each member of the team must write their own home work and hand it in with their name (underlined) and the names of other team members on the front page. Also, each member of the team must produce all required tables and graphs on his/her own, and have them identified with his/her name. Each member of the team is also required to type his/her name into the Excel Solution when handing in Homework (i.e. no hand-written names on Excel Solutions will be accepted).

1. A 1200 acre farm includes a well that has capacity of 2000 acre-feet of water per year. The farm can be used to raise wheat, alfalfa, and beef. Wheat can be sold at \$550 per ton. Alfalfa can be bought or sold at the market price of \$220 per ton and beef at \$1300 per ton. Each ton of wheat that the farmer produces requires one acre of land, \$50 of labor, and 1.5 acre-feet of water. Each ton of alfalfa that the farmer produces requires $\frac{1}{3}$ acre of land, \$40 of labor and 0.6 acre-feet of water. Each ton of beef that is produced requires 0.80 acres of land, \$50 of labor, 2 acre feet of water and 2.5 tons of alfalfa. The farmer can neither buy or sell water, and wants to run the farm to maximize the annual profit.

   (a) Formulate a linear program to help the farmer find the maximizing plan to run the farm.

   (b) Discuss if this linear model is a good approximation of the reality, i.e., discuss if the proportionality, divisibility and additivity assumptions hold for this problem.

   (c) Find the solution of this linear program using EXCEL.

2. A company makes two products in a single plant. It runs the plant for 100 hours each week. Each unit of product A that the company produces requires two hours of plant capacity, earns the company a contribution of \$1000, and causes, as an undesirable

side effect, the emission of 4 ounces of particulate matter. Each unit of product B that the company produces requires one hour of plant capacity, earns the company a contribution of $2000, and causes, as an undesirable side effect, the emission of 3 ounces of particulate matter and 1 ounce of chemicals. The EPA (Environmental Protection Agency) requires the company to limit particulate matter to at most 240 ounces per week and the chemicals to at most 60 ounces per week.

(a) Formulate a linear program to find the most profitable production plan that meets the EPA standards.

(b) Graphically, find the optimum solution to this linear program.

(c) What is the optimum solution if the profit of product B becomes $1500.

(d) Find a range of variation in the profit for product A such that the optimum solution found in (b) remains optimal. Do the same for the profit of product B.

(e) Repeat question (d) for the optimum solution found in (c).

(f) Find how much the EPA standards on the particulate matter and the chemicals can vary assuming that the optimal solution is still determined by the same binding constraints that determined the optimal solutions of (b), and of (c). Also do the same for the hours of week of operations of the plant.

(g) For both the problems of parts (b) and (c) find the shadow prices EPA standards and the weekly hours of operations. Discuss the use of this price in decision making.

(h) The company has found out that they cannot sell more than 50 units of product B. What will the new solution be?

3. As a schedule setter for an airline you must schedule exactly one early morning departure from Detroit to each of the four cities in the table below. Due to competition, the contribution earned by each flight depends on its departure time, as indicated below. For example the most profitable time for a departure to O"Hare is 7:30am. Your airline has the permission to depart at any time between 7am to 8am, but you

have only two departure gates, and you cannot schedule more than two departures at any time.

Time	Laguardia	O"Hare	Logan	National
7:00 am	8.2	7.0	5.6	9.5
7:30 am	7.8	8.2	4.4	8.8
8:00 am	6.9	7.8	3.1	7.0

(a) Formulate a linear integer model to maximize contribution.

(b) Solve this problem on EXCEL.

(c) Another airline wishes to rent one departure gate at 7:00 am. What is the smallest rent that is profitable for you to charge?

(d) It has been decided that there is not enough space to accommodate all travellers with destinations of Laguardia and O"Hare at the gates, so these flights cannot go at the same time. Reformulate this model to reflect this new constraint, and solve it on EXCEL.

4. Consider the linear program:

$$\begin{array}{lrcrcl} \text{maximize} & 2x_1 & + & 3x_2 & & \\ & x_1 & & & \leq & 6 \\ & x_1 & + & x_2 & \leq & 7 \\ & & & 2x_2 & \leq & 9 \\ & -x_1 & + & 3x_2 & \leq & 9 \\ & x_1 \geq 0 & & x_2 \geq 0 & & \end{array}$$

(a) Plot the feasible region of this linear program.

For each one of the questions below, please find the value of the objective function and the values of x_1 and x_2.

i. Solve this linear program graphically, and confirm the answer by solving on EXCEL.

ii. Change the objective function so that this problem has multiple solutions.

iii. Consider the problem with *the first, the second and the third* constraints dropped. Graphically solve this revised problem, and note that the problem is now unbounded.

iv. Change the objective function so that the modified problem has a solution.

5. Kelly Jumper is going on a holiday to New Zealand, and has heard that the New Zealanders eat almost anything, and, over the years, have built up a tremendous resistance to germs. As such, Kelly does not trust any eating establishment except McDonald's, and therefore, while on holiday, has decided to live entirely on McDonald's food. Kelly has managed to procure the information below (Table 1) from a McDonald's outlet in a remote area of New Zealand where most of the holiday will be spent. Additionally, McDonald's offers a special of 2 McChickens for $4.00 with single McChickens selling for the regular price of $2.50. Although the menu is limited compared with the McDonalds we know (and love?), there may be some information missing, e.g., soft drinks. Kelly has obtained a health brochure that contains a number of basic recommendations on daily diet, including:

   (a) Sodium: not more than 3220mg of sodium per day for adults.

   (b) Calories: about 2600 kCals for a male, and about 2000 for a female.

   (c) Fat: not more that 35% of the total calories should be from fat.

   (d) Cholesterol: not to exceed 500 milligrams of cholesterol

   (e) Protein: at least 75 grams of protein

   (f) Carbs: at least 75 grams of carbohydrates

   In addition to these general nutritional guidelines, Kelly may have considerations and preferences of her own that she would like to incorporate into the daily menu. Kelly wants to know what the least expensive diet is, consisting entirely of McDonalds foods listed above, that meets the listed nutritional criteria and other constraints. Your task is to use your knowledge of Operations Modelling to answer this question. Prepare a report for Kelly that analyzes this question. Describe all the additional assumptions you made, how you represented the nutritional requirement constraints,

what additional considerations you incorporated in your model, etc. Discuss whether the optimal solution to your model represents an acceptable daily diet. You should also consider several scenarios reflecting possible additional constraints Kelly may want to impose (e.g., what if Kelly decides to be adventurous, and wants to eat at least one Kiwiburger a day?, What if she does not want to eat beef burgers more than once a day?, etc.). Your report should clearly describe the scenarios you considered and the resulting menus and their costs and nutritional advantages/disadvantages. As an appendix, include the mathematical model(s) you constructed to derive these answers, as well as the EXCEL output with the names of the team members typed into the EXCEL output.

TABLE 1

Nutrient Content Of McDonald's Foods.

Table showing energy and nutrient content per serving.

Menu item	Energy kCalories	Fat Grams	Protein Grams	Carbo. Grams	Cholesterol Milligrams	Sodium Milligrams	Price ($)
Big Mac	452	21	26	42	11	1219	2.00
Cheeseburger	289	12	17	30	33	650	1.00
Hamburger	251	10	13	29	28	570	0.90
Kiwiburger	533	32	37	26	105	524	3.00
McFeast	496	25	32	38	64	722	2.10
Quarter Pounder	499	27	32	34	56	948	1.90
Vegetarian Salad Burger	275	7	10	45	7	612	1.60
Filet-O-Fish	324	9	15	48	5	821	2.30
McChicken	378	12	17	53	10	865	2.50
Chicken McNuggets(6)	329	21	19	17	54	804	2.50
French Fries (large)	311	16	5	39	3	175	1.50
Barbecue Sauce	42	Trace	Trace	11	0	357	free
Mustard Sauce	68	2	1	12	15	360	free
Sweet & Sour Sauce	46	Trace	Trace	12	0	285	free
Curry Sauce	46	Trace	Trace	12	0	390	free
Apple Pie	242	13	3	30	4	480	1.30
Apricot Pie	250	13	3	32	5	233	1.30
Cookies	328	14	5	48	0	125	1.40
Banana Shake	235	2	12	44	4	190	2.00
Chocolate Shake	198	3	12	32	9	232	2.00
Strawberry Shake	228	3	12	40	11	277	2.00
Vanilla Shake	127	2	11	17	11	186	2.00
Caramel Sundae	151	3	7	25	12	122	1.80
Hot Chocolate Sundae	180	5	8	27	9	144	1.90
Strawberry Sundae	184	2	6	37	7	76	1.80
Soft Serve Icecream+Cone	163	2	6	33	5	58	1.70
Orange Juice	61	0	0	16	0	38	1.85

(a) Energy KCal: This is the measure of the total available energy in a food.

(b) Carbohydrate: Carbohydrate is the most useful form of food fuel to the body and occurs as either starches or sugars in foods. 1 gram provides 3.75 kCal.

(c) Fat: Fats are found in both animal and plant foods and, although structurally different, provide the same total amount of energy. Fats are the most concentrated form of food energy available. I gram provides 9kCal.

(d) Protein: Proteins provide the essential materials for growth and repair of body tissue. l gram provides 4kCals.

(e) The information above has been calculated by an independent New Zealand laboratory & Consultant Nutritionist/Dietician.

Thanks to Shane G. Henderson for this model.