Name:_____

Section Registered In: _____

Math 125 – Homework 3

Due Wednesday, March 7

1. An auto company manufactures cars, pickup trucks, and sport utility vehicles (SUV's). The parts for the body of each vehicle must be stamped out on a press, undercoated, then finish-painted. Suppose the press line can press 35 car bodies per day if just cars are being made (that is, it requires 1/35 of a day to press one car body), 28 truck bodies per day if just trucks are being made, and 24.5 SUV bodies per day if just SUV's are being made. The undercoating line could coat 40 car bodies per day if just cars were being made, 30 trucks per day if just trucks were being made, and 24 SUV's per day if just SUV's were being made. The paint line could paint 30 car bodies per day if just cars were being made, 40 trucks per day if just trucks were being made, and 40 SUV's per day if just SUV's were being made. Assume that the rate at which the vehicles are stamped and painted does not change if a combination of vehicles are made during the day. The profit on a car, truck and SUV are \$3500, \$4500, \$5500 respectively.

(a) Set up the linear program that will maximize profits.

(b) Clearly explain why the simplex algorithm can be used to attempt to solve this linear program and form the initial simplex table.

(c) Solve the linear program.

(d) In the context of the word problem, clearly explain the meaning of the slack variables defined by the basic feasible solution of the final simplex table.

2. Let $A = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$ and $AB = \begin{bmatrix} -9 & 18 & -9 \\ 54 & -81 & 27 \end{bmatrix}$

(a) Find the dimensions of matrix B. Clearly explain your answer.

(b) Find matrix B.

Because the theorems on the properties of matrix addition, scalar multiplication and matrix multiplication look so familiar, we are tempted to think that matrices act exactly like real numbers. This is not true. For example, in the real number system, if $r^2 = 1$ then $r = \pm 1$. This same relation does not hold with matrix multiplication. In other words, $A^2 = I$ does not imply that $A = \pm I$.

3.

(a) Explain why for the product A^2 to make sense, that it is necessary that A be a square matrix.

(b) Begin with $(A - I)(A + I) = 0$. Using the properties of multiplication and addition, show that any square matrix A that satisfies the equation $(A - I)(A + I) = 0$ also satisfies the condition $A^2 = I$. Justify each algebraic simplification with the appropriate property.

(c) Find a 2×2 matrix A such that $A^2 = I$ but $A \neq \pm I$.

Again, in the real number system if $ab = 0$ then either $a = 0$, $b = 0$ or both. This is another property that does not hold in matrix multiplication.

4. Find a 2×2 matrix with all non-zero entries such that $A^2 = 0$. (Hint: Rather than a trial-and-error approach, you may want to start with the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and find the conditions on a, b, c, and d that result in $A^2 = 0$.)