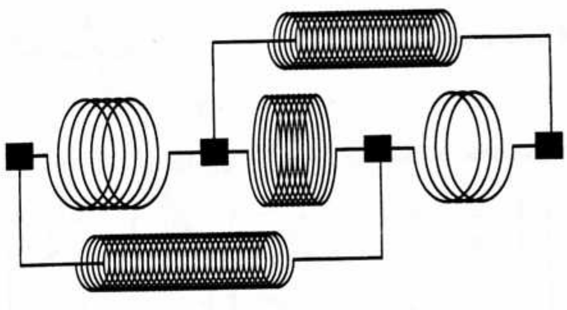

Figure 4.4: Spring-mass system with next-nearest neighbor couplings.

select any nontrivial values. Verify numerically that the force on the right wall is

$$F_{\text{rw}} = -k_0(L_{\text{w}} - L_0)$$

where $L_0 = L_1 + L_2 + L_3 + L_4$ and

$$\frac{1}{k_0} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \frac{1}{k_4}$$

This is the law of equivalent springs. Since the matrix **K** is fixed, it is more efficient to use matrix inverse rather than Gaussian elimination. [Computer]

12. Gaussian elimination is not the only way to compute the inverse of a matrix. Consider the iterative scheme,

$$\mathbf{X}_{n+1} = 2\mathbf{X}_n - \mathbf{X}_n\mathbf{A}\mathbf{X}_n$$

where $\mathbf{X}_1$ is the initial guess for $\mathbf{A}^{-1}$. Write a computer program that uses this scheme to compute the inverse of a matrix. Test your program with a few matrices, including some that are singular, and comment on your results. How good does your initial guess have to be? When might this scheme be preferable to Gaussian elimination? For more details, see Exercise 4.22. [Computer]

13. Consider the spring-mass system in Figure 4.4 (a simple model of a short polymer molecule). The blocks are free to move but only along the x-axis. The springs connecting adjacent blocks have spring constant k_1, while the two outer springs have stiffness k_2. All the springs have a rest length of one. (a) Write the matrix equation for the equilibrium positions of the blocks. [Pencil] (b) Write a program that plots the total length of the system as a function of k_1/k_2. [Computer]

14. Consider a system of coupled masses (such as in Figure 4.3) with $N-1$ blocks. The spring constants are

(a) $k_1 = k_2 = k_3 = \ldots = k_N = 1$
(b) $k_1 = 2k_2 = 3k_3 = \ldots = Nk_N = 1$
(c) $k_1 = 4k_2 = 9k_3 = \ldots = N^2k_N = 1$
(d) $k_1 = 2k_2 = 4k_3 = \ldots = 2^{(N-1)}k_N = 1$