Integration By Trigonometric Substitution: Solving Tricky Problems?
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To whom it may concern:

Hello and greetings from the USA!

I'm having trouble with finding Integrals by Trigonometric Substitution. The problems I am faced with strike me as being more tricky than the ones in either the text book they come from, or in my other math reference books. Perhaps you can offer some suggestions on how to deal with them, and their ilk.

Before continuing, allow me to take a moment to explain the term "Trigonometric Substitution" on the off chance that the term isn't universal.

"Trigonometric Substitution," is (as I understand it) a method of finding the integrals of terms involving the following:

$$\sqrt{\left(a^2 - x^2\right)} \qquad \sqrt{\left(a^2 + x^2\right)} \qquad \sqrt{\left(x^2 - a^2\right)}$$

With your continued indulgence, I shall demonstrate the procedure by integrating an equation using Trigonometric Substitution.

$$\int \frac{x^2}{\sqrt{\left(4 - x^2\right)}}\, dx \qquad x := 2\cdot\sin(\theta) \qquad dx := 2\cdot\cos(\theta)\cdot d\theta \qquad \theta := \operatorname{asin}\left(\frac{x}{2}\right)$$

$$\int \left[\frac{\left(4\cdot\sin(\theta)^2\right)}{\sqrt{\left(4 - 4\cdot\sin(\theta)^2\right)}}\right]\cdot 2\cdot\cos(\theta)\, d\theta \rightarrow \qquad \int \left[\frac{\left(4\cdot\sin(\theta)^2\right)}{\sqrt{\left[4\cdot\left(1 - \sin(\theta)^2\right)\right]}}\right]\cdot 2\cdot\cos(\theta)\, d\theta \rightarrow$$

$$\int \left[\frac{\left(4\cdot\sin(\theta)^2\right)}{\sqrt{\left[4\cdot\left(\cos(\theta)^2\right)\right]}}\right]\cdot 2\cdot\cos(\theta)\, d\theta \rightarrow \qquad \int \left[\frac{\left(4\cdot\sin(\theta)^2\right)}{2\cdot\cos(\theta)}\right]\cdot 2\cdot\cos(\theta)\, d\theta \rightarrow$$

$$4\cdot\int \sin(\theta)^2\, d\theta \rightarrow$$

N.B. Calculated by using "integration by parts" method; work is not shown

$$4\cdot\frac{-(\sin(\theta)\cdot\cos(\theta) + \theta)}{2} + C \rightarrow \qquad -2\cdot\sin(\theta)\cdot\frac{\sqrt{\left[4\cdot\left(1 - \sin(\theta)^2\right)\right]}}{\sqrt{4}} + 2\,\theta + C \rightarrow$$

Substituting in reverse eventually gives the answer:

$$-x\cdot\frac{\sqrt{\left(4 - x^2\right)}}{2} + 2\cdot\operatorname{asin}\left(\frac{x}{2}\right) + C$$

Of course, any corrections or suggestions would be most welcome!

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Now that I've shown you what I can do, here are some examples of what I can't do:

$$\int \left[\frac{\sqrt{\left(1+x^2\right)}}{x}\right] dx \qquad \int \frac{1}{\left(x^2+9\right)^2}\, dx \qquad \int \frac{1}{\left(x\cdot\sqrt{16-9x^2}\right)}\, dx \qquad \int x^2\cdot\sqrt{\left[(1-x)^2\right]}\, dx$$

$$\int \sqrt{\left[1+\left(\frac{1}{x}\right)^2\right]}\, dx$$

N.B. It is how to solve problems like these, rather than the answers to these problems, that I'd like to learn. If, however, you'd like to use any of these problems as an example, you may feel free to do so.

My attempts to solve the preceding problems terminate, after a considerable amount of work, with an interim result that either defies reverse-substitution, or that would require unrealistic amounts of "integration by parts" to solve. This, in turn, makes me wonder if I haven't gone astray in my calculations, or if Dr. Mendelson (the author of my text book, "Beginning Calculus") is really the sadistic bastard I believe him to be. <g>

Since Dr. M does not discuss hyperbolic functions in his book, they should not be necessary in order to resolve any of these problems. Then again, one of the reasons I so loathe Dr. M is that he will include, among his "supplemental problems", problems that require information he did not previously supply.

Before bothering you, I have consulted several math books. Unfortunately, every math book I consulted discussed only relatively simple examples of this sort. If you know of a particularly well written book or web site that would be helpful, please, mention it.

In conclusion, thanks in advance for your assistance!

Best regards,
Richard Kanarek