Problem:

Let V be a vector space of all real continuous function on closed interval [-1, 1].
Let Wo be a set of all odd functions in V and let We be a set of all even functions in V.

(i) **Show** that Wo and We are subspaces and then show that $V = W_o \oplus W_e$.
(ii) **Find** a projection mapping onto Wo parallel to We and projection mapping onto We parallel to Wo.
(iii) Let L: V -> V be a mapping that transforms *f* from V into function that is given by $L(f)(x) := \int_0^x f(t)\,dt.$ **Show** that L is linear mapping and **state** whether the following is true or false:

$$L[W_o] \subset W_e \quad \text{and} \quad L[W_e] \subset W_o$$

Solution:

(i) $V = W_o \oplus W_e$ then every element in V can be written as

$$f(x) = \frac{1}{2}\Big(\underbrace{f(x) + f(-x)}_{even\ \ function}\Big) + \frac{1}{2}\Big(\underbrace{f(x) - f(-x)}_{odd\ \ function}\Big)$$

$$\{function\} = (even\ \ function) \oplus (odd\ \ function)$$

(ii) To find a projection mapping we use:
N and P are R –submodules of M such that $M = N \oplus P$ and φ that is projection mapping onto N parallel to P. Then

1. $N = \operatorname{Im}\varphi = \{x \in M \mid \varphi(x)\}$
2. $Ker\varphi = P$
3. $\varphi \circ \varphi = \varphi$