For further details concerning the physics involved and animations of the trajectories of the particles, see the following web sites:

- lompado.uah.edu/Links/CrossedFields.html
- www.phy.ntnu.edu.tw/java/emField/emField.html
- www.physics.ucla.edu/plasma-exp/Beam/

13.1 Exercises

1–2 Find the domain of the vector function.

1. $\mathbf{r}(t) = \langle t^2, \sqrt{t-1}, \sqrt{5-t} \rangle$

2. $\mathbf{r}(t) = \dfrac{t-2}{t+2}\,\mathbf{i} + \sin t\,\mathbf{j} + \ln(9-t^2)\,\mathbf{k}$

3–6 Find the limit.

3. $\lim_{t\to 0^+} \langle \cos t, \sin t, t \ln t \rangle$

4. $\lim_{t\to 0} \left\langle \dfrac{e^t - 1}{t}, \dfrac{\sqrt{1+t} - 1}{t}, \dfrac{3}{1+t} \right\rangle$

5. $\lim_{t\to 1} \left(\sqrt{t+3}\,\mathbf{i} + \dfrac{t-1}{t^2-1}\,\mathbf{j} + \dfrac{\tan t}{t}\,\mathbf{k} \right)$

6. $\lim_{t\to \infty} \left\langle \arctan t, e^{-2t}, \dfrac{\ln t}{t} \right\rangle$

7–14 Sketch the curve with the given vector equation. Indicate with an arrow the direction in which t increases.

7. $\mathbf{r}(t) = \langle t^4 + 1, t \rangle$

8. $\mathbf{r}(t) = \langle t^3, t^2 \rangle$

9. $\mathbf{r}(t) = \langle t, \cos 2t, \sin 2t \rangle$

10. $\mathbf{r}(t) = \langle 1 + t, 3t, -t \rangle$

11. $\mathbf{r}(t) = \langle \sin t, 3, \cos t \rangle$

12. $\mathbf{r}(t) = t\,\mathbf{i} + t\,\mathbf{j} + \cos t\,\mathbf{k}$

13. $\mathbf{r}(t) = t^2\,\mathbf{i} + t^4\,\mathbf{j} + t^6\,\mathbf{k}$

14. $\mathbf{r}(t) = \sin t\,\mathbf{i} + \sin t\,\mathbf{j} + \sqrt{2}\cos t\,\mathbf{k}$

15–18 Find a vector equation and parametric equations for the line segment that joins P to Q.

15. $P(0, 0, 0)$, $Q(1, 2, 3)$

16. $P(1, 0, 1)$, $Q(2, 3, 1)$

17. $P(1, -1, 2)$, $Q(4, 1, 7)$

18. $P(-2, 4, 0)$, $Q(6, -1, 2)$

19–24 Match the parametric equations with the graphs (labeled I–VI). Give reasons for your choices.

19. $x = \cos 4t, \quad y = t, \quad z = \sin 4t$

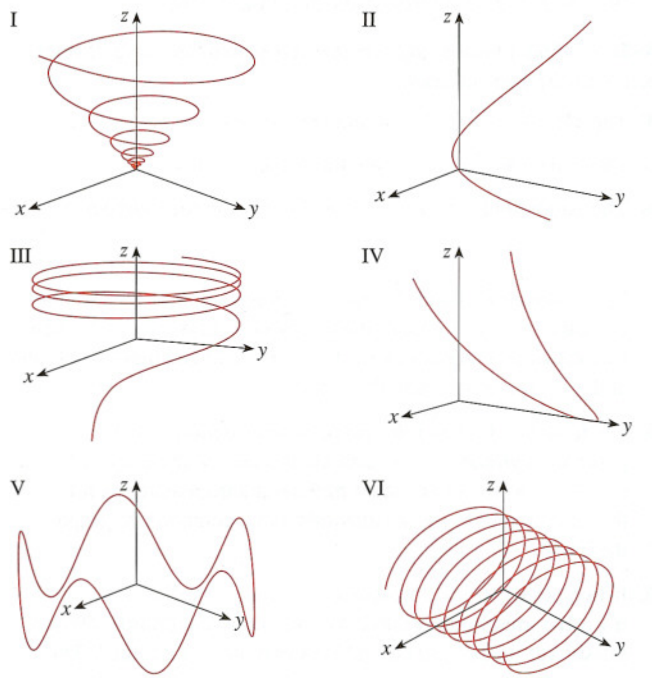
20. $x = t, \quad y = t^2, \quad z = e^{-t}$

21. $x = t, \quad y = 1/(1 + t^2), \quad z = t^2$

22. $x = e^{-t} \cos 10t, \quad y = e^{-t} \sin 10t, \quad z = e^{-t}$

23. $x = \cos t, \quad y = \sin t, \quad z = \sin 5t$

24. $x = \cos t, \quad y = \sin t, \quad z = \ln t$



25. Show that the curve with parametric equations $x = t \cos t$, $y = t \sin t$, $z = t$ lies on the cone $z^2 = x^2 + y^2$, and use this fact to help sketch the curve.

26. Show that the curve with parametric equations $x = \sin t$, $y = \cos t$, $z = \sin^2 t$ is the curve of intersection of the surfaces $z = x^2$ and $x^2 + y^2 = 1$. Use this fact to help sketch the curve.

27–30 Use a computer to graph the curve with the given vector equation. Make sure you choose a parameter domain and viewpoints that reveal the true nature of the curve.

27. $\mathbf{r}(t) = \langle \sin t, \cos t, t^2 \rangle$

28. $\mathbf{r}(t) = \langle t^4 - t^2 + 1, t, t^2 \rangle$

29. $\mathbf{r}(t) = \langle t^2, \sqrt{t-1}, \sqrt{5-t} \rangle$

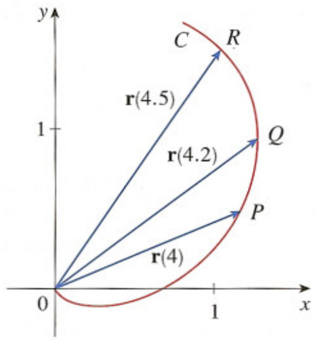
30. $\mathbf{r}(t) = \langle \sin t, \sin 2t, \sin 3t \rangle$

13.2 Exercises

1. The figure shows a curve C given by a vector function $\mathbf{r}(t)$.
(a) Draw the vectors $\mathbf{r}(4.5) - \mathbf{r}(4)$ and $\mathbf{r}(4.2) - \mathbf{r}(4)$.
(b) Draw the vectors

$$\frac{\mathbf{r}(4.5) - \mathbf{r}(4)}{0.5} \quad \text{and} \quad \frac{\mathbf{r}(4.2) - \mathbf{r}(4)}{0.2}$$

(c) Write expressions for $\mathbf{r}'(4)$ and the unit tangent vector $\mathbf{T}(4)$.
(d) Draw the vector $\mathbf{T}(4)$.



2. (a) Make a large sketch of the curve described by the vector function $\mathbf{r}(t) = \langle t^2, t \rangle$, $0 \leqslant t \leqslant 2$, and draw the vectors $\mathbf{r}(1)$, $\mathbf{r}(1.1)$, and $\mathbf{r}(1.1) - \mathbf{r}(1)$.
(b) Draw the vector $\mathbf{r}'(1)$ starting at (1, 1) and compare it with the vector

$$\frac{\mathbf{r}(1.1) - \mathbf{r}(1)}{0.1}$$

Explain why these vectors are so close to each other in length and direction.

3–8 |||
(a) Sketch the plane curve with the given vector equation.
(b) Find $\mathbf{r}'(t)$.
(c) Sketch the position vector $\mathbf{r}(t)$ and the tangent vector $\mathbf{r}'(t)$ for the given value of t.

3. $\mathbf{r}(t) = \langle \cos t, \sin t \rangle$, $t = \pi/4$

4. $\mathbf{r}(t) = \langle 1 + t, \sqrt{t} \rangle$, $t = 1$

5. $\mathbf{r}(t) = (1 + t)\,\mathbf{i} + t^2\mathbf{j}$, $t = 1$

6. $\mathbf{r}(t) = e^t\,\mathbf{i} + e^{-t}\,\mathbf{j}$, $t = 0$

7. $\mathbf{r}(t) = e^t\mathbf{i} + e^{3t}\mathbf{j}$, $t = 0$

8. $\mathbf{r}(t) = 2 \sin t\,\mathbf{i} + 3 \cos t\,\mathbf{j}$, $t = \pi/3$

9–16 ||| Find the derivative of the vector function.

9. $\mathbf{r}(t) = \langle t^2, 1 - t, \sqrt{t} \rangle$

10. $\mathbf{r}(t) = \langle \cos 3t, t, \sin 3t \rangle$

11. $\mathbf{r}(t) = \mathbf{i} - \mathbf{j} + e^{4t}\,\mathbf{k}$

12. $\mathbf{r}(t) = \sin^{-1}t\,\mathbf{i} + \sqrt{1 - t^2}\,\mathbf{j} + \mathbf{k}$

13. $\mathbf{r}(t) = e^{t^2}\mathbf{i} - \mathbf{j} + \ln(1 + 3t)\,\mathbf{k}$

14. $\mathbf{r}(t) = at \cos 3t\,\mathbf{i} + b \sin^3 t\,\mathbf{j} + c \cos^3 t\,\mathbf{k}$

15. $\mathbf{r}(t) = \mathbf{a} + t\,\mathbf{b} + t^2\mathbf{c}$

16. $\mathbf{r}(t) = t\,\mathbf{a} \times (\mathbf{b} + t\,\mathbf{c})$

17–20 ||| Find the unit tangent vector $\mathbf{T}(t)$ at the point with the given value of the parameter t.

17. $\mathbf{r}(t) = \langle 6t^5, 4t^3, 2t \rangle$, $t = 1$

18. $\mathbf{r}(t) = 4\sqrt{t}\,\mathbf{i} + t^2\mathbf{j} + t\,\mathbf{k}$, $t = 1$

19. $\mathbf{r}(t) = \cos t\,\mathbf{i} + 3t\,\mathbf{j} + 2 \sin 2t\,\mathbf{k}$, $t = 0$

20. $\mathbf{r}(t) = 2 \sin t\,\mathbf{i} + 2 \cos t\,\mathbf{j} + \tan t\,\mathbf{k}$, $t = \pi/4$

21. If $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$, find $\mathbf{r}'(t)$, $\mathbf{T}(1)$, $\mathbf{r}''(t)$, and $\mathbf{r}'(t) \times \mathbf{r}''(t)$.

22. If $\mathbf{r}(t) = \langle e^{2t}, e^{-2t}, te^{2t} \rangle$, find $\mathbf{T}(0)$, $\mathbf{r}''(0)$, and $\mathbf{r}'(t) \cdot \mathbf{r}''(t)$.

23–26 ||| Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point.

23. $x = t^5$, $y = t^4$, $z = t^3$; (1, 1, 1)

24. $x = t^2 - 1$, $y = t^2 + 1$, $z = t + 1$; (−1, 1, 1)

25. $x = e^{-t} \cos t$, $y = e^{-t} \sin t$, $z = e^{-t}$; (1, 0, 1)

26. $x = \ln t$, $y = 2\sqrt{t}$, $z = t^2$; (0, 2, 1)

27–28 ||| Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point. Illustrate by graphing both the curve and the tangent line on a common screen.

27. $x = t$, $y = \sqrt{2} \cos t$, $z = \sqrt{2} \sin t$; $(\pi/4, 1, 1)$

28. $x = \cos t$, $y = 3e^{2t}$, $z = 3e^{-2t}$; (1, 3, 3)

29. Determine whether the curve is smooth.
(a) $\mathbf{r}(t) = \langle t^3, t^4, t^5 \rangle$
(b) $\mathbf{r}(t) = \langle t^3 + t, t^4, t^5 \rangle$
(c) $\mathbf{r}(t) = \langle \cos^3 t, \sin^3 t \rangle$

30. (a) Find the point of intersection of the tangent lines to the curve $\mathbf{r}(t) = \langle \sin \pi t, 2 \sin \pi t, \cos \pi t \rangle$ at the points where $t = 0$ and $t = 0.5$.
(b) Illustrate by graphing the curve and both tangent lines.

31. The curves $\mathbf{r}_1(t) = \langle t, t^2, t^3 \rangle$ and $\mathbf{r}_2(t) = \langle \sin t, \sin 2t, t \rangle$ intersect at the origin. Find their angle of intersection correct to the nearest degree.

32. At what point do the curves $\mathbf{r}_1(t) = \langle t, 1 - t, 3 + t^2 \rangle$ and $\mathbf{r}_2(s) = \langle 3 - s, s - 2, s^2 \rangle$ intersect? Find their angle of intersection correct to the nearest degree.