## 12.7 Exercises

**1.** What are cylindrical coordinates? For what types of surfaces do they provide convenient descriptions?

**2.** What are spherical coordinates? For what types of surfaces do they provide convenient descriptions?

**3–8** Plot the point whose cylindrical coordinates are given. Then find the rectangular coordinates of the point.

**3.** $(2, \pi/4, 1)$

**4.** $(1, 3\pi/2, 2)$

**5.** $(3, 0, -6)$

**6.** $(1, \pi, e)$

**7.** $(4, -\pi/3, 5)$

**8.** $(5, \pi/6, 6)$

**9–12** Change from rectangular to cylindrical coordinates.

**9.** $(1, -1, 4)$

**10.** $(3, 3, -2)$

**11.** $(-1, -\sqrt{3}, 2)$

**12.** $(3, 4, 5)$

**13–18** Plot the point whose spherical coordinates are given. Then find the rectangular coordinates of the point.

**13.** $(1, 0, 0)$

**14.** $(3, 0, \pi)$

**15.** $(1, \pi/6, \pi/6)$

**16.** $(5, \pi, \pi/2)$

**17.** $(2, \pi/3, \pi/4)$

**18.** $(2, \pi/4, \pi/3)$

**19–22** Change from rectangular to spherical coordinates.

**19.** $(1, \sqrt{3}, 2\sqrt{3})$

**20.** $(0, \sqrt{3}, 1)$

**21.** $(0, -1, -1)$

**22.** $(-1, 1, \sqrt{6})$

**23–26** Change from cylindrical to spherical coordinates.

**23.** $(1, \pi/6, \sqrt{3})$

**24.** $(\sqrt{6}, \pi/4, \sqrt{2})$

**25.** $(\sqrt{3}, \pi/2, -1)$

**26.** $(4, \pi/8, 3)$

**27–30** Change from spherical to cylindrical coordinates.

**27.** $(2, 0, 0)$

**28.** $(2\sqrt{2}, 3\pi/2, \pi/2)$

**29.** $(8, \pi/6, \pi/2)$

**30.** $(4, \pi/4, \pi/3)$

**31–36** Describe in words the surface whose equation is given.

**31.** $r = 3$

**32.** $\rho = 3$

**33.** $\phi = 0$

**34.** $\phi = \pi/2$

**35.** $\phi = \pi/3$

**36.** $\theta = \pi/3$

**37–48** Identify the surface whose equation is given.

**37.** $z = r^2$

**38.** $r = 4 \sin\theta$

**39.** $\rho \cos\phi = 2$

**40.** $\rho \sin\phi = 2$

**41.** $r = 2\cos\theta$

**42.** $\rho = 2\cos\phi$

**43.** $r^2 + z^2 = 25$

**44.** $r^2 - 2z^2 = 4$

**45.** $\rho^2(\sin^2\phi \cos^2\theta + \cos^2\phi) = 4$

**46.** $\rho^2(\sin^2\phi - 4\cos^2\phi) = 1$

**47.** $r^2 = r$

**48.** $\rho^2 - 6\rho + 8 = 0$

**49–56** Write the equation (a) in cylindrical coordinates and (b) in spherical coordinates.

**49.** $z = x^2 + y^2$

**50.** $x^2 + y^2 + z^2 = 2$

**51.** $x = 3$

**52.** $x^2 + y^2 + z^2 + 2z = 0$

**53.** $x^2 - y^2 - 2z^2 = 4$

**54.** $y^2 + z^2 = 1$

**55.** $x^2 + y^2 = 2y$

**56.** $z = x^2 - y^2$

**57–62** Sketch the solid described by the given inequalities.

**57.** $r^2 \leqslant z \leqslant 2 - r^2$

**58.** $0 \leqslant \theta \leqslant \pi/2, \quad r \leqslant z \leqslant 2$

**59.** $\rho \leqslant 2, \quad 0 \leqslant \phi \leqslant \pi/2, \quad 0 \leqslant \theta \leqslant \pi/2$

**60.** $2 \leqslant \rho \leqslant 3, \quad \pi/2 \leqslant \phi \leqslant \pi$

**61.** $-\pi/2 \leqslant \theta \leqslant \pi/2, \quad 0 \leqslant \phi \leqslant \pi/6, \quad 0 \leqslant \rho \leqslant \sec\phi$

**62.** $0 \leqslant \phi \leqslant \pi/3, \quad \rho \leqslant 2$

**63.** A cylindrical shell is 20 cm long, with inner radius 6 cm and outer radius 7 cm. Write inequalities that describe the shell in an appropriate coordinate system. Explain how you have positioned the coordinate system with respect to the shell.

**64.** (a) Find inequalities that describe a hollow ball with diameter 30 cm and thickness 0.5 cm. Explain how you have positioned the coordinate system that you have chosen.
(b) Suppose the ball is cut in half. Write inequalities that describe one of the halves.

**65.** A solid lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$. Write a description of the solid in terms of inequalities involving spherical coordinates.

**66.** Use a graphing device to draw the solid enclosed by the paraboloids $z = x^2 + y^2$ and $z = 5 - x^2 - y^2$.

**67.** Use a graphing device to draw a silo consisting of a cylinder with radius 3 and height 10 surmounted by a hemisphere.

**68.** The latitude and longitude of a point $P$ in the Northern Hemisphere are related to spherical coordinates $\rho$, $\theta$, $\phi$ as follows. We take the origin to be the center of the Earth and the positive $z$-axis to pass through the North Pole. The positive $x$-axis passes through the point where the prime meridian (the meridian through Greenwich, England) intersects the equator. Then the latitude of $P$ is $\alpha = 90° - \phi°$ and the longitude is $\beta = 360° - \theta°$. Find the great-circle distance from Los Angeles (lat. 34.06° N, long. 118.25° W) to Montréal (lat. 45.50° N, long. 73.60° W). Take the radius of the Earth to be 3960 mi. (A *great circle* is the circle of intersection of a sphere and a plane through the center of the sphere.)