Exercise 5-1

A *Graph each inequality in Problems 1–10.*

1. $y \le x - 1$ **2.** $y > x + 1$

3. $3x - 2y > 6$ **4.** $2x - 5y \le 10$

5. $x \ge -4$ **6.** $y < 5$

7. $6x + 4y \ge 24$ **8.** $4x + 8y \ge 32$

9. $5x \le -2y$ **10.** $6x \ge 4y$

In Problems 11–14,

(A) graph the set of points that satisfy the inequality.

(B) graph the set of points that do not satisfy the inequality.

11. $2x + 3y < 18$ **12.** $3x + 4y > 24$

13. $5x - 2y \ge 20$ **14.** $3x - 5y \le 30$

In Problems 15–18, match the solution region of each system of linear inequalities with one of the four regions shown in the figure.

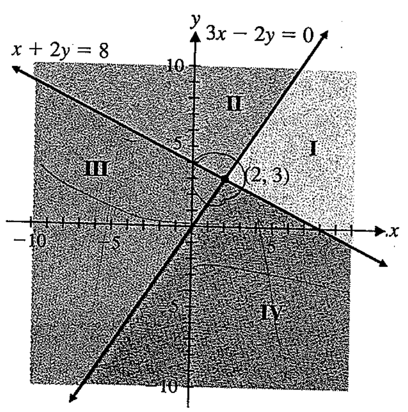


Figure for 15–18

15. $x + 2y \le 8$, $3x - 2y \ge 0$ **16.** $x + 2y \ge 8$, $3x - 2y \le 0$

17. $x + 2y \ge 8$, $3x - 2y \ge 0$ **18.** $x + 2y \le 8$, $3x - 2y \le 0$

In Problems 19–22, solve each system of linear inequalities graphically.

19. $3x + y \ge 6$, $x \le 4$ **20.** $3x + 4y \le 12$, $y \ge -3$

21. $x - 2y \le 12$, $2x + y \ge 4$ **22.** $2x + 5y \le 20$, $x - 5y \ge -5$

In Problems 23–26, solve each system two ways (see Explore–Discuss 2):

(A) by shading the points that satisfy each inequality in the system

(B) by shading the points that do not satisfy each inequality in the system

In each case, explain how you can recognize the solution region.

23. $x + y \le 5$, $2x - y \le 1$ **24.** $x - 2y \le 1$, $x + 3y \ge 12$

25. $2x + y \ge 4$, $3x - y \le 7$ **26.** $3x + y \ge -2$, $x - 2y \ge -6$

B *In Problems 27–30, match the solution region of each system of linear inequalities with one of the four regions shown in the figure. Identify the corner points of each solution region.*

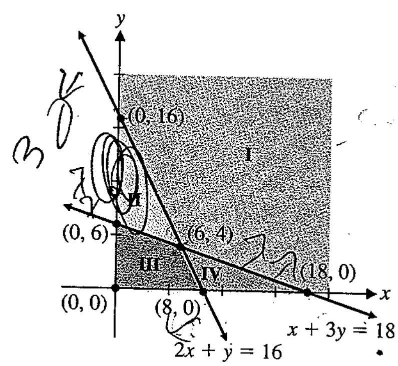


Figure for 27–30

27. $x + 3y \le 18$, $2x + y \ge 16$, $x \ge 0$, $y \ge 0$ **28.** $x + 3y \le 18$, $2x + y \le 16$, $x \ge 0$, $y \ge 0$

29. $x + 3y \ge 18$, $2x + y \ge 16$, $x \ge 0$, $y \ge 0$ **30.** $x + 3y \ge 18$, $2x + y \le 16$, $x \ge 0$, $y \ge 0$

Solve the systems in Problems 31–40 graphically, and indicate whether each solution region is bounded or unbounded. Find the coordinates of each corner point.

31. $2x + 3y \le 12$, $x \ge 0$, $y \ge 0$ **32.** $3x + 4y \le 24$, $x \ge 0$, $y \ge 0$

33. $$\begin{aligned} 2x + y &\le 10 \\ x + 2y &\le 8 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$

34. $$\begin{aligned} 6x + 3y &\le 24 \\ 3x + 6y &\le 30 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$

35. $$\begin{aligned} 2x + y &\ge 10 \\ x + 2y &\ge 8 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$

36. $$\begin{aligned} 4x + 3y &\ge 24 \\ 3x + 4y &\ge 8 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$

37. $$\begin{aligned} 2x + y &\le 10 \\ x + y &\le 7 \\ x + 2y &\le 12 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$

38. $$\begin{aligned} 3x + y &\le 21 \\ x + y &\le 9 \\ x + 3y &\le 21 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$

39. $$\begin{aligned} 2x + y &\ge 16 \\ x + y &\ge 12 \\ x + 2y &\ge 14 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$

40. $$\begin{aligned} 3x + y &\ge 24 \\ x + y &\ge 16 \\ x + 3y &\ge 30 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$

C *Solve the systems in Problems 41–50 graphically, and indicate whether each solution region is bounded or unbounded. Find the coordinates of each corner point.*

41. $$\begin{aligned} x + 4y &\le 32 \\ 3x + y &\le 30 \\ 4x + 5y &\ge 51 \end{aligned}$$

42. $$\begin{aligned} x + y &\le 11 \\ x + 5y &\ge 15 \\ 2x + y &\ge 12 \end{aligned}$$

43. $$\begin{aligned} 4x + 3y &\le 48 \\ 2x + y &\ge 24 \\ x &\le 9 \end{aligned}$$

44. $$\begin{aligned} 2x + 3y &\ge 24 \\ x + 3y &\le 15 \\ y &\ge 4 \end{aligned}$$

45. $$\begin{aligned} x - y &\le 0 \\ 2x - y &\le 4 \\ 0 \le x &\le 8 \end{aligned}$$

46. $$\begin{aligned} 2x + 3y &\ge 12 \\ -x + 3y &\le 3 \\ 0 \le y &\le 5 \end{aligned}$$

47. $$\begin{aligned} -x + 3y &\ge 1 \\ 5x - y &\ge 9 \\ x + y &\le 9 \\ x &\le 5 \end{aligned}$$

48. $$\begin{aligned} x + y &\le 10 \\ 5x + 3y &\ge 15 \\ -2x + 3y &\le 15 \\ 2x - 5y &\le 6 \end{aligned}$$

49. $$\begin{aligned} 16x + 13y &\le 120 \\ 3x + 4y &\ge 25 \\ -4x + 3y &\le 11 \end{aligned}$$

50. $$\begin{aligned} 2x + 2y &\le 21 \\ -10x + 5y &\le 24 \\ 3x + 5y &\ge 37 \end{aligned}$$

Problems 51 and 52 introduce an algebraic process for finding the corner points of a solution region without drawing a graph. We will have a great deal more to say about this process later in this chapter.

51. Consider the following system of inequalities and corresponding boundary lines:

$$\begin{aligned} 3x + 4y &\le 36 \qquad & 3x + 4y &= 36 \\ 3x + 2y &\le 30 & 3x + 2y &= 30 \\ x &\ge 0 & x &= 0 \\ y &\ge 0 & y &= 0 \end{aligned}$$

(A) Use algebraic methods to find the intersection points (if any exist) for each possible pair of boundary lines. (There are six different possible pairs.)

(B) Test each intersection point in all four inequalities to determine which are corner points.

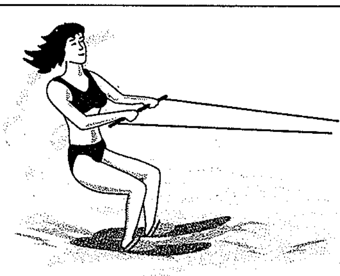
52. Repeat Problem 51 for

$$\begin{aligned} 2x + y &\le 16 \qquad & 2x + y &= 16 \\ 2x + 3y &\le 36 & 2x + 3y &= 36 \\ x &\ge 0 & x &= 0 \\ y &\ge 0 & y &= 0 \end{aligned}$$

Applications

Business & Economics

53. *Manufacturing: resource allocation.* A manufacturing company makes two types of water skis, a trick ski and a slalom ski. The trick ski requires 6 labor-hours for fabricating and 1 labor-hour for finishing. The slalom ski requires 4 labor-hours for fabricating and 1 labor-hour for finishing. The maximum labor-hours available per day for fabricating and finishing are 108 and 24, respectively. If x is the number of trick skis and y is the number of slalom skis produced per day, write a system of linear inequalities that indicates appropriate restraints on x and y. Find the set of feasible solutions graphically for the number of each type of ski that can be produced.



54. *Manufacturing: resource allocation.* A furniture manufacturing company manufactures dining room tables and chairs. A table requires 8 labor-hours for assembling and 2 labor-hours for finishing. A chair requires 2 labor-hours for assembling and 1 labor-hour for finishing. The maximum labor-hours available per day for assembly and finishing are 400 and 120, respectively. If x is the number of tables and y is the number of chairs produced per day, write a system of linear inequalities that indicates appropriate restraints on x and y. Find the set of feasible solutions graphically for the number of tables and chairs that can be produced.

55. *Manufacturing: resource allocation.* Refer to Problem 53. The company makes a profit of \$50 on each trick ski and a profit of \$60 on each slalom ski.

    (A) If the company makes 10 trick skis and 10 slalom skis per day, the daily profit will be \$1,100. Are there other production schedules that will result in a daily profit of \$1,100? How are these schedules related to the graph of the line $50x + 60y = 1{,}100$?

    (B) Find a production schedule that will produce a daily profit greater than \$1,100 and repeat part (A) for this schedule.

    (C) Discuss methods for using lines like those in parts (A) and (B) to find the largest possible daily profit.

56. *Manufacturing: resource allocation.* Refer to Problem 54. The company makes a profit of \$50 on each table and a profit of \$15 on each chair.

    (A) If the company makes 20 tables and 20 chairs per day, the daily profit will be \$1,300. Are there other production schedules that will result in a daily profit of \$1,300? How are these schedules related to the graph of the line $50x + 15y = 1{,}300$?

    (B) Find a production schedule that will produce a daily profit greater than \$1,300 and repeat part (A) for this schedule.

    (C) Discuss methods for using lines like those in parts (A) and (B) to find the largest possible daily profit.

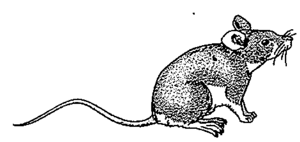
Life Sciences

57. *Nutrition: plants.* A farmer can buy two types of plant food, mix A and mix B. Each cubic yard of mix A contains 20 pounds of phosphoric acid, 30 pounds of nitrogen, and 5 pounds of potash. Each cubic yard of mix B contains 10 pounds of phosphoric acid, 30 pounds of nitrogen, and 10 pounds of potash. The minimum monthly requirements are 460 pounds of phosphoric acid, 960 pounds of nitrogen, and 220 pounds of potash. If x is the number of cubic yards of mix A used and y is the number of cubic yards of mix B used, write a system of linear inequalities that indicates appropriate restraints on x and y. Find the set of feasible solutions graphically for the amounts of mix A and mix B that can be used.

58. *Nutrition: people.* A dietitian in a hospital is to arrange a special diet using two foods. Each ounce o food M contains 30 units of calcium, 10 units of iron and 10 units of vitamin A. Each ounce of food N contains 10 units of calcium, 10 units of iron, and 30 units of vitamin A. The minimum requirements i the diet are 360 units of calcium, 160 units of iron, and 240 units of vitamin A. If x is the number of ounces of food M used and y is the number of ounces of food N used, write a system of linear inequalities that reflects the conditions indicated. Fin the set of feasible solutions graphically for the amount of each kind of food that can be used.

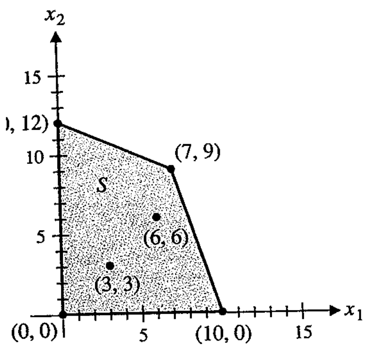
Social Sciences

59. *Psychology.* In an experiment on conditioning, a psychologist uses two types of Skinner (conditioning) boxes with mice and rats. Each mouse spends 10 minutes per day in box A and 20 minutes per da in box B. Each rat spends 20 minutes per day in box A and 10 minutes per day in box B. The total maximum time available per day is 800 minutes fo box A and 640 minutes for box B. We are intereste in the various numbers of mice and rats that can b used in the experiment under the conditions state If we let x be the number of mice used and y the number of rats used, write a system of linear inequ ities that indicates appropriate restrictions on x an y. Find the set of feasible solutions graphically.



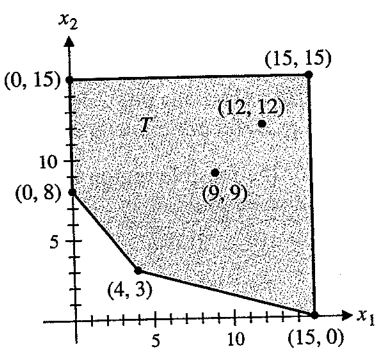
ercise 5-2

Problems 1–4, graph the isoprofit lines through (3, 3) d also through (6, 6). Use a straight edge to identify e corner point where the maximum profit occurs (see xplore–Discuss 1). Confirm your answer by construct- g a corner point table.



1. $P = x_1 + x_2$
2. $P = 4x_1 + x_2$
3. $P = 3x_1 + 7x_2$
4. $P = 9x_1 + 3x_2$

In Problems 5–8, graph the isocost lines through (9, 9) and also through (12, 12). Use a straight edge to identify the corner point where the minimum cost occurs. Confirm your answer by constructing a corner point table.



5. $C = 7x_1 + 4x_2$
6. $C = 7x_1 + 9x_2$
7. $C = 3x_1 + 8x_2$
8. $C = 2x_1 + 11x_2$

Solve the linear programming problems stated in Problems 9–26.

9. Maximize $P = 5x_1 + 5x_2$
subject to
$$2x_1 + x_2 \le 10$$
$$x_1 + 2x_2 \le 8$$
$$x_1, x_2 \ge 0$$

10. Maximize $P = 3x_1 + 2x_2$
subject to
$$6x_1 + 3x_2 \le 24$$
$$3x_1 + 6x_2 \le 30$$
$$x_1, x_2 \ge 0$$

11. Minimize and maximize
$z = 2x_1 + 3x_2$
subject to
$$2x_1 + x_2 \ge 10$$
$$x_1 + 2x_2 \ge 8$$
$$x_1, x_2 \ge 0$$

12. Minimize and maximize
$z = 8x_1 + 7x_2$
subject to
$$4x_1 + 3x_2 \ge 24$$
$$3x_1 + 4x_2 \ge 8$$
$$x_1, x_2 \ge 0$$

13. Maximize $P = 30x_1 + 40x_2$
subject to
$$2x_1 + x_2 \le 10$$
$$x_1 + x_2 \le 7$$
$$x_1 + 2x_2 \le 12$$
$$x_1, x_2 \ge 0$$

14. Maximize $P = 20x_1 + 10x_2$
subject to
$$3x_1 + x_2 \le 21$$
$$x_1 + x_2 \le 9$$
$$x_1 + 3x_2 \le 21$$
$$x_1, x_2 \ge 0$$

15. Minimize and maximize
$z = 10x_1 + 30x_2$
subject to
$$2x_1 + x_2 \ge 16$$
$$x_1 + x_2 \ge 12$$
$$x_1 + 2x_2 \ge 14$$
$$x_1, x_2 \ge 0$$

16. Minimize and maximize
$z = 400x_1 + 100x_2$
subject to
$$3x_1 + x_2 \ge 24$$
$$x_1 + x_2 \ge 16$$
$$x_1 + 3x_2 \ge 30$$
$$x_1, x_2 \ge 0$$

17. Minimize and maximize
$P = 30x_1 + 10x_2$
subject to
$$2x_1 + 2x_2 \ge 4$$
$$6x_1 + 4x_2 \le 36$$
$$2x_1 + x_2 \le 10$$
$$x_1, x_2 \ge 0$$

18. Minimize and maximize
$P = 2x_1 + x_2$
subject to
$$x_1 + x_2 \ge 2$$
$$6x_1 + 4x_2 \le 36$$
$$4x_1 + 2x_2 \le 20$$
$$x_1, x_2 \ge 0$$

19. Minimize and maximize

$P = 3x_1 + 5x_2$

subject to
$$\begin{aligned} x_1 + 2x_2 &\le 6 \\ x_1 + x_2 &\le 4 \\ 2x_1 + 3x_2 &\ge 12 \\ x_1, x_2 &\ge 0 \end{aligned}$$

20. Minimize and maximize

$P = -x_1 + 3x_2$

subject to
$$\begin{aligned} 2x_1 - x_2 &\ge 4 \\ -x_1 + 2x_2 &\le 4 \\ x_2 &\le 6 \\ x_1, x_2 &\ge 0 \end{aligned}$$

21. Minimize and maximize

$P = 20x_1 + 10x_2$

subject to
$$\begin{aligned} 2x_1 + 3x_2 &\ge 30 \\ 2x_1 + x_2 &\le 26 \\ -2x_1 + 5x_2 &\le 34 \\ x_1, x_2 &\ge 0 \end{aligned}$$

22. Minimize and maximize

$P = 12x_1 + 14x_2$

subject to
$$\begin{aligned} -2x_1 + x_2 &\ge 6 \\ x_1 + x_2 &\le 15 \\ 3x_1 - x_2 &\ge 0 \\ x_1, x_2 &\ge 0 \end{aligned}$$

23. Maximize $P = 20x_1 + 30x_2$

subject to
$$\begin{aligned} 0.6x_1 + 1.2x_2 &\le 960 \\ 0.03x_1 + 0.04x_2 &\le 36 \\ 0.3x_1 + 0.2x_2 &\le 270 \\ x_1, x_2 &\ge 0 \end{aligned}$$

24. Minimize $C = 30x_1 + 10x_2$

subject to
$$\begin{aligned} 1.8x_1 + 0.9x_2 &\ge 270 \\ 0.3x_1 + 0.2x_2 &\ge 54 \\ 0.01x_1 + 0.03x_2 &\ge 3.9 \\ x_1, x_2 &\ge 0 \end{aligned}$$

25. Maximize $P = 525x_1 + 478x_2$

subject to
$$\begin{aligned} 275x_1 + 322x_2 &\le 3{,}381 \\ 350x_1 + 340x_2 &\le 3{,}762 \\ 425x_1 + 306x_2 &\le 4{,}114 \\ x_1, x_2 &\ge 0 \end{aligned}$$

26. Maximize $P = 300x_1 + 460x_2$

subject to
$$\begin{aligned} 245x_1 + 452x_2 &\le 4{,}181 \\ 290x_1 + 379x_2 &\le 3{,}888 \\ 390x_1 + 299x_2 &\le 4{,}407 \\ x_1, x_2 &\ge 0 \end{aligned}$$

In Problems 27 and 28, explain why Theorem 2 cannot be used to conclude that a maximum or minimum value exists. Graph the feasible regions and use graphs of the objective function $z = x_1 - x_2$ for various values of z to discuss the existence of a maximum value and a minimum value.

27. Minimize and maximize

$z = x_1 - x_2$

subject to
$$\begin{aligned} x_1 - 2x_2 &\le 0 \\ 2x_1 - x_2 &\le 6 \\ x_1, x_2 &\ge 0 \end{aligned}$$

28. Minimize and maximize

$z = x_1 - x_2$

subject to
$$\begin{aligned} x_1 - 2x_2 &\ge -6 \\ 2x_1 - x_2 &\ge 0 \\ x_1, x_2 &\ge 0 \end{aligned}$$

C

29. The corner points for the bounded feasible region determined by the system of linear inequalities
$$\begin{aligned} x_1 + 2x_2 &\le 10 \\ 3x_1 + x_2 &\le 15 \\ x_1, x_2 &\ge 0 \end{aligned}$$
are $O = (0, 0)$, $A = (0, 5)$, $B = (4, 3)$, and $C = (5, 0)$. If $P = ax_1 + bx_2$ and $a, b > 0$, determine conditions on a and b that will ensure that the maximum value of P occurs:

(A) Only at A
(B) Only at B
(C) Only at C
(D) At both A and B
(E) At both B and C

30. The corner points for the feasible region determined by the system of linear inequalities
$$\begin{aligned} x_1 + 4x_2 &\ge 30 \\ 3x_1 + x_2 &\ge 24 \\ x_1, x_2 &\ge 0 \end{aligned}$$
are $A = (0, 24)$, $B = (6, 6)$, and $D = (30, 0)$. If $C = ax_1 + bx_2$ and $a, b > 0$, determine conditions on a and b that will ensure that the minimum value of C occurs:

(A) Only at A
(B) Only at B
(C) Only at D
(D) At both A and B
(E) At both B and D

Applications

In Problems 31–46, construct a mathematical model in the form of a linear programming problem. (The answers in the back of the book for these application problems include the model.) Then solve by the geometric method.

Business & Economics

31. *Manufacturing: resource allocation.* A manufacturing company makes two types of water skis, a trick ski and a slalom ski. The relevant manufacturing data are given in the table.

Department	Labor-Hours per Ski: Trick Ski	Labor-Hours per Ski: Slalom Ski	Maximum Labor-Hours Available per Day
Fabricating	6	4	108
Finishing	1	1	24

(A) If the profit on a trick ski is \$40 and the profit on a slalom ski is \$30, how many of each type of ski should be manufactured each day to realize a maximum profit? What is the maximum profit?

(B) Discuss the effect on the production schedule and the maximum profit if the profit on a slalom ski decreases to \$25.

(C) Discuss the effect on the production schedule and the maximum profit if the profit on a slalom ski increases to \$45.

32. *Manufacturing: resource allocation.* A furniture manufacturing company manufactures dining room tables and chairs. The relevant manufacturing data are given in the table.

Department	Labor-Hours per Unit: Table	Labor-Hours per Unit: Chair	Maximum Labor-Hours Available per Day
Assembly	8	2	400
Finishing	2	1	120
Profit per unit	\$90	\$25	

(A) How many tables and chairs should be manufactured each day to realize a maximum profit? What is the maximum profit?

(B) Discuss the effect on the production schedule and the maximum profit if the marketing department of the company decides that the number of chairs produced should be at least four times the number of tables produced.

33. *Manufacturing: production scheduling.* A furniture company has two plants that produce the lumber used in manufacturing tables and chairs. In 1 day of operation, plant A can produce the lumber required to manufacture 20 tables and 60 chairs, and plant B can produce the lumber required to manufacture 25 tables and 50 chairs. The company needs enough lumber to manufacture at least 200 tables and 500 chairs.

(A) If it costs \$1,000 to operate plant A for 1 day and \$900 to operate plant B for 1 day, how many days should each plant be operated to produce a sufficient amount of lumber at a minimum cost? What is the minimum cost?

(B) Discuss the effect on the operating schedule and the minimum cost if the daily cost of operating plant A is reduced to \$600 and all other data in part (A) remain the same.

(C) Discuss the effect on the operating schedule and the minimum cost if the daily cost of operating plant B is reduced to \$800 and all other data in part (A) remain the same.

34. *Manufacturing: resource allocation.* An electronics firm manufactures two types of personal computers, a standard model and a portable model. The production of a standard computer requires a capital expenditure of \$400 and 40 hours of labor. The production of a portable computer requires a capital expenditure of \$250 and 30 hours of labor. The firm has \$20,000 capital and 2,160 labor-hours available for production of standard and portable computers.

(A) What is the maximum number of computers the company is capable of producing?

(B) If each standard computer contributes a profit of \$320 and each portable model contributes a profit of \$220, how much profit will the company make by producing the maximum number of computers determined in part (A)? Is this the maximum profit? If not, what is the maximum profit?

35. *Transportation.* The officers of a high school senior class are planning to rent buses and vans for a class trip. Each bus can transport 40 students, requires 3 chaperones, and costs \$1,200 to rent. Each van can transport 8 students, requires 1 chaperone, and costs \$100 to rent. Since there are 400 students in the senior class that may be eligible to go on the trip, the officers must plan to accommodate at least 400 students. Since only 36 parents have volunteered to serve as chaperones, the officers must plan to use at most 36 chaperones. How many vehicles of each type should the officers rent in order to minimize the transportation costs? What are the minimal transportation costs?

36. *Transportation.* Refer to Problem 35. If each van can transport 7 people and there are 35 available chaperones, show that the optimal solution found graphically involves decimals. Find all feasible solutions with integer coordinates and identify the one that minimizes the transportation costs. Can this optimal integer solution be obtained by rounding the optimal decimal solution? Explain.

37. *Investment.* An investor has $60,000 to invest in a CD and a mutual fund. The CD yields 5% and the mutual fund yields on the average 9%. The mutual fund requires a minimum investment of $10,000 and the investor requires that twice as much should be invested in CDs as in the mutual fund. How much should be invested in CDs and how much in the mutual fund to maximize the return? What is the maximum return?

38. *Investment.* An investor has $24,000 to invest in bonds of AAA and B qualities. The AAA bonds yield on the average 6% and the B bonds yield 10%. The investor requires that at least three times as much money should be invested in AAA bonds as in B bonds. How much should be invested in each type of bond to maximize the return? What is the maximum return?

39. *Pollution control.* Because of new federal regulations on pollution, a chemical plant introduced a new, more expensive process to supplement or replace an older process used in the production of a particular chemical. The older process emitted 20 grams of sulfur dioxide and 40 grams of particulate matter into the atmosphere for each gallon of chemical produced. The new process emits 5 grams of sulfur dioxide and 20 grams of particulate matter for each gallon produced. The company makes a profit of 60¢ per gallon and 20¢ per gallon on the old and new processes, respectively.

(A) If the government allows the plant to emit no more than 16,000 grams of sulfur dioxide and 30,000 grams of particulate matter daily, how many gallons of the chemical should be produced by each process to maximize daily profit? What is the maximum daily profit?

(B) Discuss the effect on the production schedule and the maximum profit if the government decides to restrict emissions of sulfur dioxide to 11,500 grams daily and all other data remain unchanged.

(C) Discuss the effect on the production schedule and the maximum profit if the government decides to restrict emissions of sulfur dioxide to 7,200 grams daily and all other data remain unchanged.

40. *Capital expansion.* A fast-food chain plans to expand by opening several new restaurants. The chain operates two types of restaurants, drive-through and full-service. A drive-through restaurant costs $100,000 to construct, requires 5 employees, and has an expected annual revenue of $200,000. A full-service restaurant costs $150,000 to construct, requires 15 employees, and has an expected annual revenue of $500,000. The chain has $2,400,000 in capital available for expansion. Labor contracts require that they hire no more than 210 employees, and licensing restrictions require that they open no more than 20 new restaurants. How many restaurants of each type should the chain open in order to maximize the expected revenue? What is the maximum expected revenue? How much of their capital will they use and how many employees will they hire?

Life Sciences

41. *Nutrition: plants.* A fruit grower can use two types of fertilizer in his orange grove, brand *A* and brand *B*. The amounts (in pounds) of nitrogen, phosphoric acid, and chloride in a bag of each brand are given in the table. Tests indicate that the grove needs at least 1,000 pounds of phosphoric acid and at most 400 pounds of chloride.

	Pounds per Bag	
	Brand *A*	**Brand *B***
Nitrogen	8	3
Phosphoric acid	4	4
Chloride	2	1

(A) If the grower wants to maximize the amount of nitrogen added to the grove, how many bags of each mix should be used? How much nitrogen will be added?

(B) If the grower wants to minimize the amount of nitrogen added to the grove, how many bags of each mix should be used? How much nitrogen will be added?

42. *Nutrition: people.* A dietitian in a hospital is to arrange a special diet composed of two foods, *M* and *N*. Each ounce of food *M* contains 30 units of calcium, 10 units of iron, 10 units of vitamin A, and 8 units of cholesterol. Each ounce of food *N* contains 10 units of calcium, 10 units of iron, 30 units of vitamin A, and 4 units of cholesterol. If the minimum daily requirements are 360 units of calcium, 160 units of iron, and 240 units of vitamin A, how many ounces of each food should be used to meet the minimum requirements and at the same time minimize the cholesterol intake? What is the minimum cholesterol intake?

43. *Nutrition: plants.* A farmer can buy two types of plant food, mix *A* and mix *B*. Each cubic yard of mix *A*