Example 4: Use Gaussian elimination and the method of backward substitution to solve the following linear system:

$$\begin{aligned} -2x_1 + 6x_2 - 4x_3 &= -6 \\ 5x_1 + 8x_2 + 3x_3 &= 8 \\ 4x_1 - 3x_2 + x_3 &= 2 \end{aligned}$$

$$\left[\begin{array}{ccc|c} -2 & 6 & -4 & -6 \\ 5 & 8 & 3 & 8 \\ 4 & -3 & 1 & 2 \end{array}\right] \quad R_1' = -\frac{1}{2}R_1$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 3 \\ 5_{-5} & 8_{15} & 3_{-10} & 8_{-15} \\ 4_{-4} & -3_{12} & 1_{-8} & 2_{-12} \end{array}\right] \quad \begin{array}{l} R_2' = R_2 - 5R_1 \\ R_3' = R_3 - 4R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 3 \\ 0 & 23 & -7 & -7 \\ 0 & 9 & -7 & -10 \end{array}\right] \quad R_2' = \frac{1}{23}R_2$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 3 \\ 0 & 1 & -7/23 & -7/23 \\ 0 & 9_{-9} & -7_{\ 63/23} & -10_{\ 63/23} \end{array}\right] \quad R_3' = R_3 - 9R_2$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 3 \\ 0 & 1 & -7/23 & -7/23 \\ 0 & 0 & -98/23 & -167/23 \end{array}\right] \quad R_3' = -\frac{23}{98}R_3$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 3 \\ 0 & 1 & -7/23 & -7/23 \\ 0 & 0 & 1 & 167/98 \end{array}\right]$$

Now in row echelon form.

$$\begin{aligned} x_1 - 3x_2 + 2x_3 &= 3. \\ x_2 - \frac{7}{23}x_3 &= \frac{-7}{23} \\ x_3 &= \frac{167}{98} \end{aligned}$$

Back substitution.

$$x_2 - \frac{7}{23}\left(\frac{167}{98}\right) = \frac{-7}{23}$$

$$x_2 = \frac{3}{4}$$

$$x_1 - 3\left(\frac{3}{4}\right) + 2\left(\frac{167}{98}\right) = 3$$

$$x_1 = \frac{23}{98}$$

Sol'n point

$$\left(\frac{23}{98}, \frac{3}{4}, \frac{167}{98}\right).$$

Rmk: if you don't fractions, you will not get correct answers.