

Name: _____

Section Registered In: _____

Math 125 – Homework 2

Due Friday, February 17

1. The three elementary row operations come from a very natural place – they are the matrix equivalent of the same three operations allowed when reducing and solving a system of equations. Look at Lesson 2.1, Example 3 on page 17 in the notes. Clearly explain, matrix by matrix, how the row operations correspond to operations on the underlying system of equations.

2. A computer manufacturer produces three models of hardware: a desktop, a laptop, and a server. The production times for the desktop are 0.5 hours of assembly and 0.1 of packaging. For the laptop, it takes 1.0 hours of assembly and 0.6 hours of packaging. The server requires 1.5 hours of assembly and 1.2 hours of packaging. Suppose there are 75 labor-hours available in assembly and 54 labor hours available in packaging. Find all possible production schedules. That is, find all possible combinations of the three types of computers which can be manufactured using all the available labor-hours in both areas. Recall that it is hard to sell a partially built computer. Do all row operations by hand.

3.

(a) Show that if $ad - bc \neq 0$ then the reduced row echelon form of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Assume that $a \neq 0$.

(b) Use your answer to part (a) to show that the linear system

$$ax + by = k$$

$$cx + dy = l$$

has exactly one solution when $ad - bc \neq 0$.

(c) If $ad - bc = 0$, under what conditions will the system have a solution? When will there be no solution?

4. A company 403(b) plan allows employees to divide their investments among three mutual funds: an aggressive growth fund, a hybrid fund, and an income fund. The aggressive growth fund currently owns 90% stocks, 5% bonds, and 5% cash. The hybrid fund owns 60% stocks, 20% bonds, and 20% cash. The income fund owns 25% stocks, 50% bonds, and 25% cash. Suppose an employee has \$10,000 to place in her 403(b) this year. For each of the following investment proportions, tell whether it is possible for her to divide the \$10,000 among the three funds so that the total amount invested in stocks, bonds, and cash are exactly in her desired proportions. If such an investment is possible, tell how much money should be invested in each of the three funds. Use a multisystem to do both parts simultaneously. *You may use your calculator to move the augmented matrix into rref form. You do not need to show the row operations.*

- (a) 85% stocks, 10% bonds, and 5% cash
- (b) 45% stocks, 35% bonds, and 20% cash

5. In general, a matrix's row echelon form can vary a bit. A matrix's reduced row echelon form is always unique. In other words, there is only one specific reduced row echelon form matrix associated with each matrix.

- (a) Consider the following homogeneous system:

$$\begin{aligned} 2x_1 - x_2 + x_4 + 4x_5 &= 0 \\ 2x_1 - 2x_2 + x_3 + 4x_4 - 3x_5 &= 0 \\ 2x_1 - 4x_2 + x_3 + 6x_4 + 6x_5 &= 0 \end{aligned}$$

Write this system as an augmented matrix, *use your calculator* and find the row echelon form, and the reduced row echelon form.

- (b) Consider the following homogeneous system:

$$\begin{aligned} 2x_1 - 2x_2 + x_3 + 4x_4 - 3x_5 &= 0 \\ 2x_1 - x_2 + x_4 + 4x_5 &= 0 \\ 2x_1 - 4x_2 + x_3 + 6x_4 + 6x_5 &= 0 \end{aligned}$$

Write this system as an augmented matrix. *Use your calculator* and find the row echelon form and the reduced row echelon form. How is this system different than the first? What

row operation would move the augmented matrix from part (a) to this one?

(c) Consider the following homogeneous system:

$$2x_1 - 4x_2 + x_3 + 6x_4 + 6x_5 = 0$$

$$2x_1 - x_2 + x_4 + 4x_5 = 0$$

$$2x_1 - 2x_2 + x_3 + 4x_4 - 3x_5 = 0$$

Write this system as an augmented matrix. *Use your calculator* and find the row echelon form and the reduced row echelon form. How is this system different than the first? What row operation would move the augmented matrix from part (a) to this one?

(d) Explain why the row echelon forms found in parts (b) and (c) are alternative forms of the row echelon form matrix found in part (a).

(e) Comparing the three different row echelon forms, how are they different? What is the same about them? What does this suggest about row echelon form of any matrix?

(f) How are the reduced row echelon forms different for the above augmented matrices? What does this suggest about the reduced row echelon form on any matrix?

(g) While we are here, solve the homogeneous system.