Suppose that $G$ is a finite group and $H$ is a normal subgroup of$ G$.

1. Explain briefly how the quotient group $G/H$ is defined. [You should explain what the elements of $G/H$ are, how the multiplication is defined, what the identity element is and how inverses are defined. You do *not* need to prove that the multiplication is well-defined.]

Show that the map
 $π:G\rightarrow G/H$

 $π\left(g\right)=gH$

 is a homomorphism. What is the kernel of$ π$?

1. Hence show that if $ρ:G/H\rightarrow GL(n, C)$ is a representation of the quotient group $G/H$ then the map $\tilde{ρ}:G\rightarrow GL(n, C)$ defined by

$$\tilde{ρ}\left(g\right)=ρ(gH)$$

(for each$ g\in G$) is a representation of$ G$.

1. Now let$ G=A\_{4}$, the alternating group of even permutations of$ \left\{1, 2, 3, 4\right\}$. Let
$$H=\left\{e, \left(12\right)\left(34\right), \left(13\right)\left(24\right), (14)(23)\right\}$$

Show that$ H⊴G$. To which well-known group is the quotient group $G/H$ isomorphic?

1. For $G$ and $H$ as in (iii) above, write down (without proof) three degree one representations of$ G/H$.
2. Hence describe three degree 1 (and hence irreducible) representations of$ G=A\_{4}$. Write down the characters of the representations you have found on representatives of the four conjugacy classes of$ G$.