

4. (a) Let $A_i(x)$ be the components of a rank $\binom{0}{1}$ tensor field on a smooth n dimensional manifold in a chart with coordinates $\{x^i\}$. Do the quantities

$$W_{ij}(x) = \frac{\partial}{\partial x^i} A_j(x)$$

transform as a rank $\binom{0}{2}$ tensor field under an arbitrary coordinate transformation?

(b) Same question, but replace the $W_{ij}(x)$ in part (a) with

$$F_{ij} = \frac{\partial}{\partial x^i} A_j(x) - \frac{\partial}{\partial x^j} A_i(x)$$

5. Show that under a coordinate tangent $y^i = f^i(x)$ that the components w_i of a 1-form transform as

$$w_i(y) = \frac{\partial x^i}{\partial y^i} w_i(x).$$

Hint: use the fact that the contraction $w_i v^i$ is a scalar.